Canonical Functional (Holomorphic) Quantization of Pseudo-Photons in Planar Systems

Pedro Castelo Ferreira

CENTRA, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

$\texttt{hep}-\texttt{th}/\texttt{0609239}$	1. Maxwell Action
$\texttt{hep}-\texttt{th}/\texttt{0703193}$	2. Extended $U_e(1)\times U_g(1)$ Electromagnetism
$\texttt{hep}-\texttt{th}/\texttt{0703194}$	3. Functional Quantization

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\nabla \cdot \mathbf{E} &= \rho_e \\
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... derived phenomenologically and unified in 1861.

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$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad , \quad J_e^\nu = (\rho_e, \mathbf{j}_e)$$

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Equations of Motion: $\dfrac{\delta S}{\delta A_\mu} = 0$

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Regularity of Gauge Fields $\Rightarrow$ Bianchi Identities

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Usual definitions: $E^i = F^{0i} \quad , \quad B^i = \frac{1}{2}\epsilon^{ijk}F_{jk}$

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$\Rightarrow$ Hence fails to describe the full Maxwell Equations at variational level

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either the Dirac string of Wu-Yang fiber-bundle

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– although magnetic monopoles are the only justification for quantization of electric charge have so far not been detected

– however non-regular external electromagnetic fields are common in many physical systems and there is plenty of experimental evidence of violation of the Bianchi Identities

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$$eg = 2\pi\,\hbar\,n$$

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EOM A : $\partial_\mu \left(F^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho} \right) = J_e^\nu \Leftrightarrow \begin{cases} \nabla \cdot \mathbf{E} = \rho_e \\ \nabla \times \mathbf{B} - \dot{\mathbf{E}} = \mathbf{j}_e \end{cases}$

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\end{aligned}
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EOM A : $\partial_\mu\left(F^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho}\right) = J_e^\nu \;\Leftrightarrow\; \left\{\begin{array}{rcl} \nabla\cdot\mathbf{E} &=& \rho_e \\ \nabla\times\mathbf{B} - \dot{\mathbf{E}} &=& \mathbf{j}_e \end{array}\right.$

EOM C : $\partial_\mu\left(G^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}\right) = J_g^\nu \;\Leftrightarrow\; \left\{\begin{array}{rcl} \nabla\cdot\mathbf{B} &=& \rho_g \\ \nabla\times\mathbf{E} + \dot{\mathbf{B}} &=& \mathbf{j}_g \end{array}\right.$

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$$S = \frac{1}{4} \int dx^4 \left[-F_{\mu\nu} F^{\mu\nu} \right.$$

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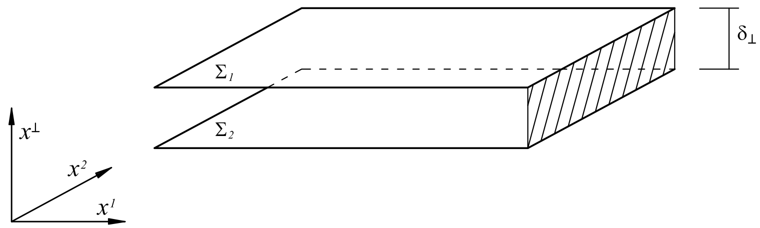

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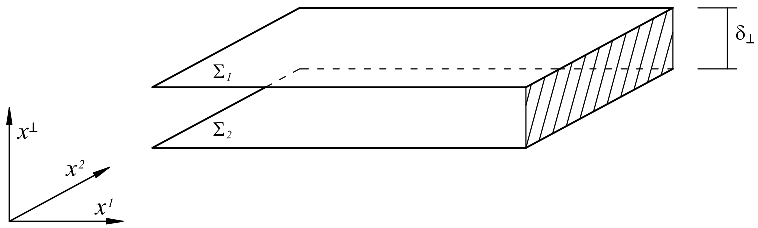

$$I, J = \mu, \perp \quad ; \quad \mu = 0, i, j \quad ; \quad i, j = 1, 2$$

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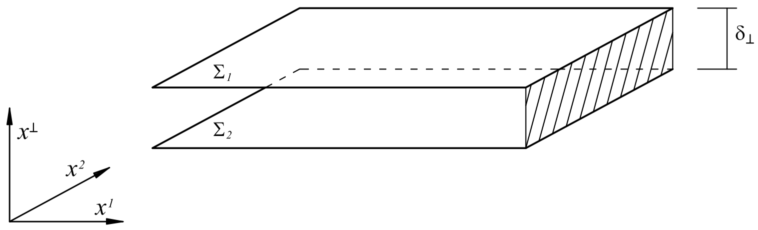

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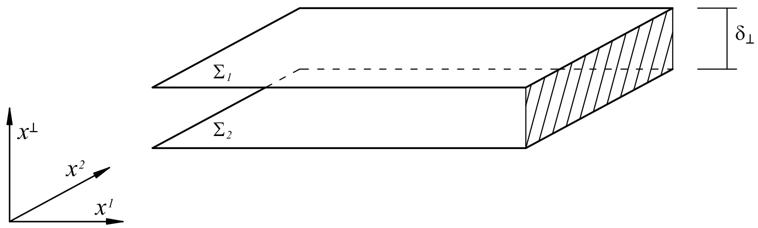

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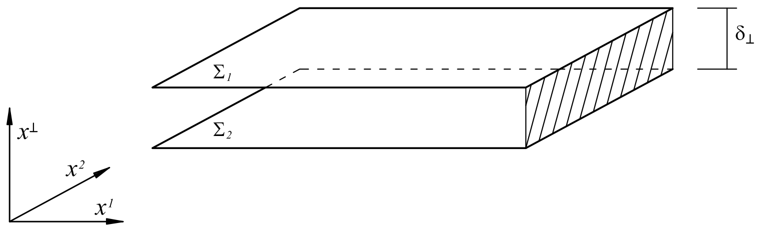

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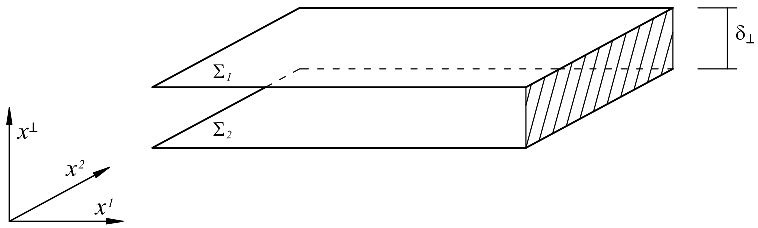

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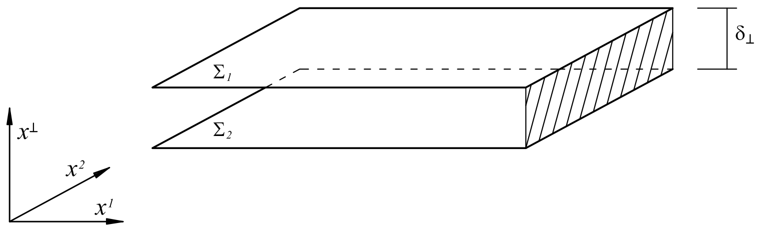

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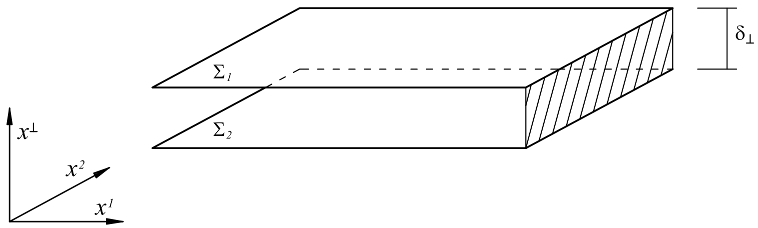

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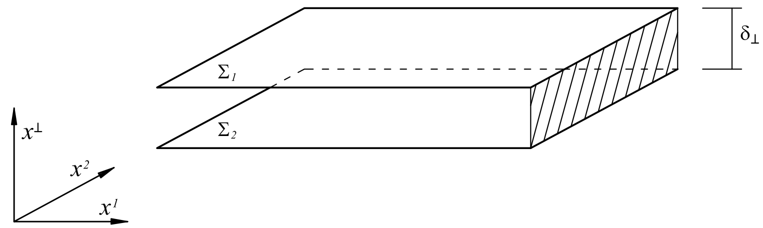

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Identification of boundaries: $\Sigma(x^\perp = 0) \cong \Sigma_1 \cong \Sigma_2$

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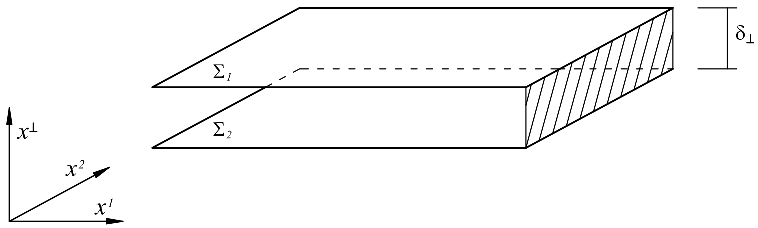

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Id. of bound.: $\displaystyle\int_{\Sigma_1 - \Sigma_2} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu C_\lambda \equiv \frac{k}{2} \int_\Sigma \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu C_\lambda$

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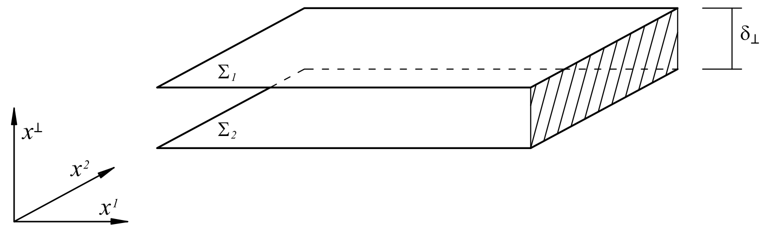

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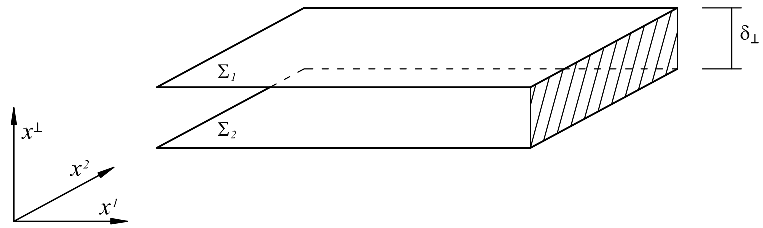

$$\mathcal{L}_4 = -\frac{1}{4} F_{IJ} F^{IJ} + \frac{1}{4} G_{IJ} G^{IJ} + \frac{1}{4} \epsilon^{IJKL} F_{IJ} G_{KL} + A_I \, J_e^I$$

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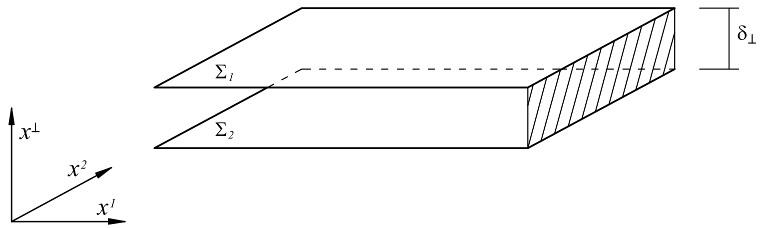

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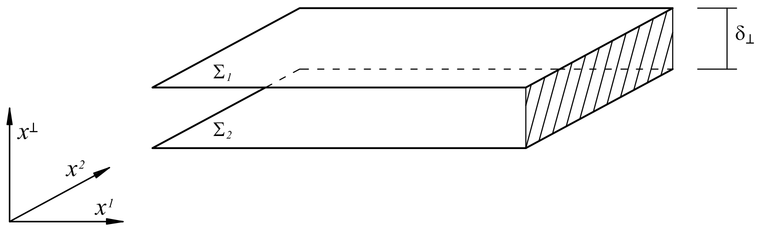

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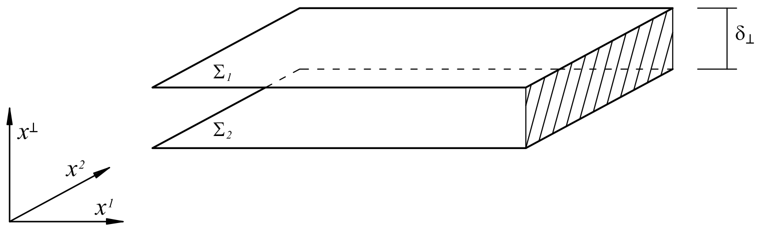

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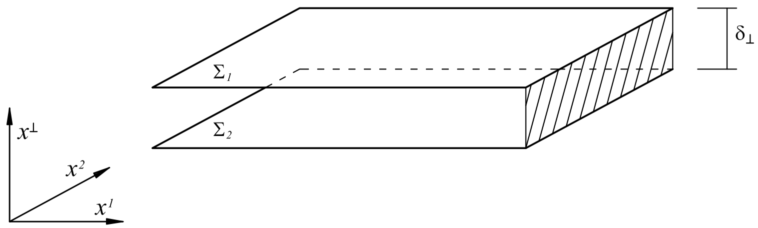

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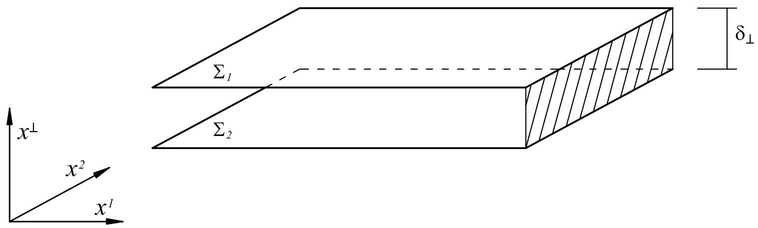

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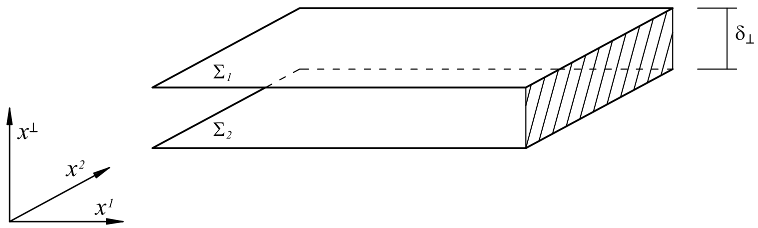

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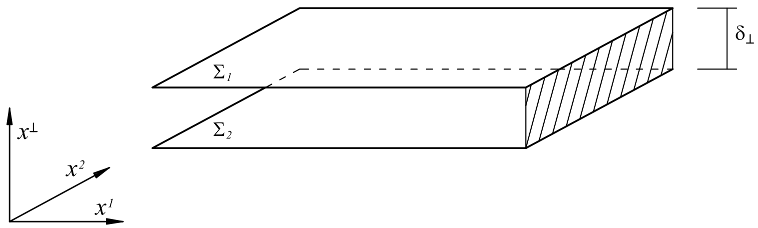

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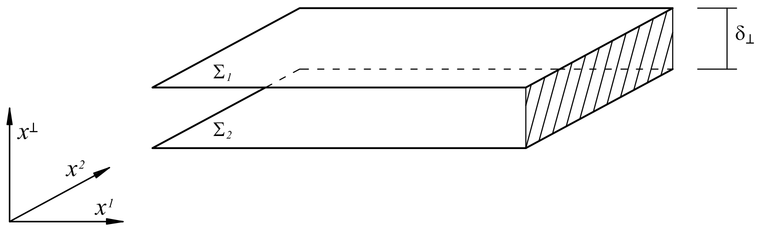

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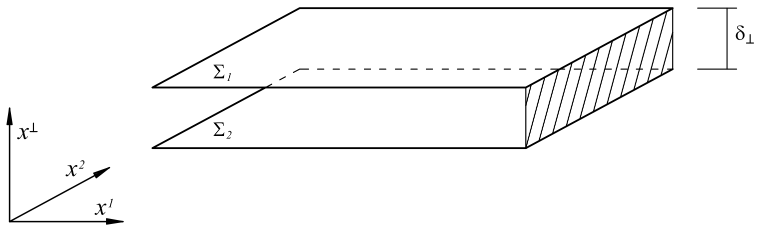

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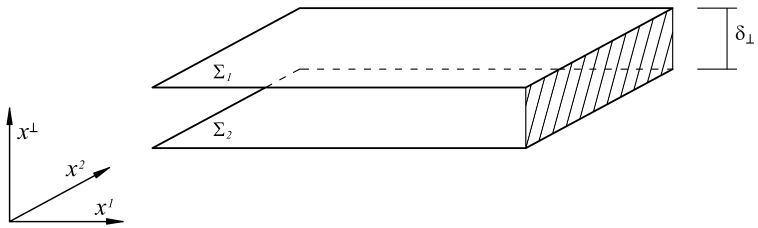

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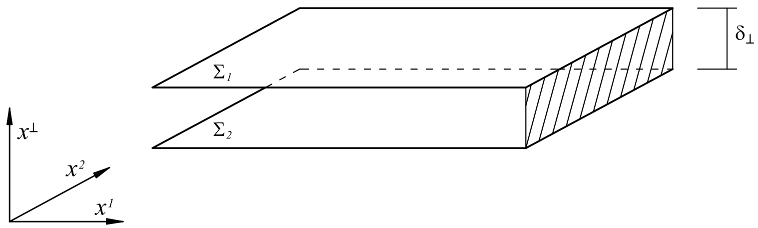

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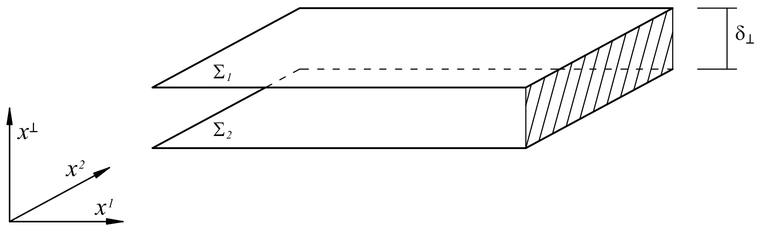

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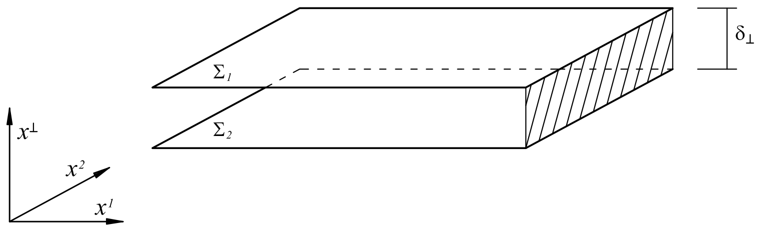

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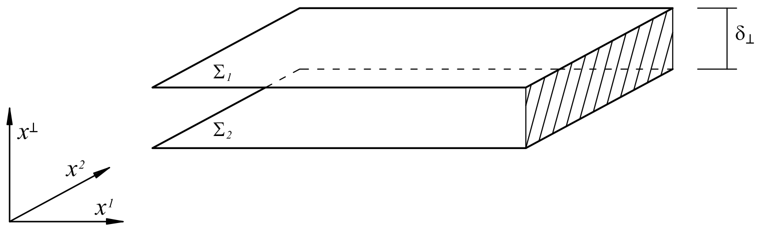

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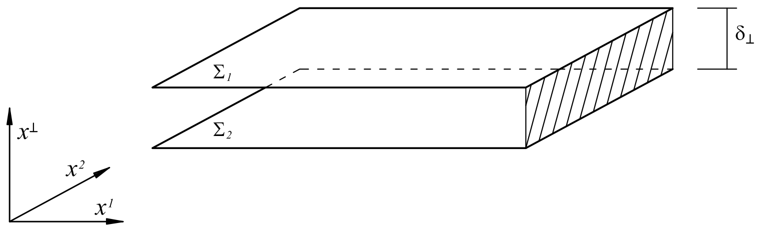

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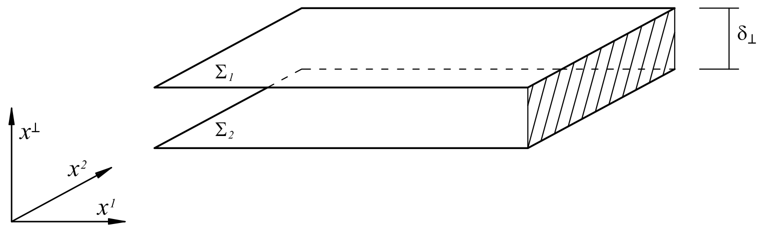

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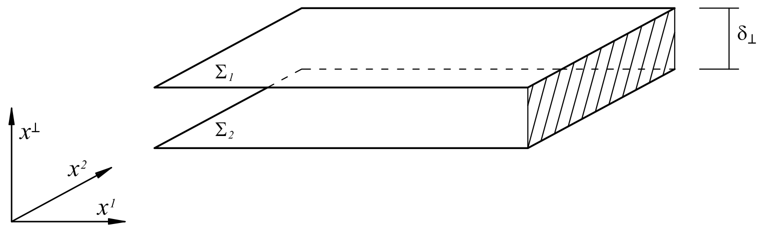

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- In planar systems we will obtain new observable consequences when considering Extended $U_e(1) \times U_g(1)$ Electromagnetism.

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α: electric vortex density , $\hat{\beta}$: magnetic vortex density

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- theoretical justification for the low energy contribution to Laughlin's wave function solutions due to the negative energy contributions of pseudo-photon excitations (which are ghost or phantoms).

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- theoretical justification for the orthogonal electric potential due to pseudo-photon electric vortexes which may justify the experimental existence of BEC condensates in bi-layer electron-electron Hall systems instead of its existence in electron-hole Hall systems as originally expected.

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$$\hat{\mathcal{G}}_A\Phi_{(0,0)}[A,C] = \left[\partial_i\left(i\frac{\delta}{\delta A_i} - \frac{k}{4\delta_\perp}\epsilon^{ij}C_j\right)\right]\Phi_{(0,0)}[A,C] = 0$$

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$$\pi^i_A = -i\,\frac{\delta}{\delta A_i} \qquad\qquad \pi^i_C = +i\,\frac{\delta}{\delta C_i}$$

we obtain 3 functional constraints:

$$\hat{\mathcal{G}}_A\Phi_{(0,0)}[A,C] = \left[\partial_i\left(i\frac{\delta}{\delta A_i} - \frac{k}{4\delta_\perp}\epsilon^{ij}C_j\right)\right]\Phi_{(0,0)}[A,C] = 0$$

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3. Functional Quantization in Planar Systems

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$$\begin{aligned}
\hat{\mathcal{H}}_{AC} \Phi_{(0,0)}[A,C] &= \Bigg[+\frac{1}{2}\left(i\frac{\delta}{\delta A_i} + \frac{k}{4\delta_\perp}\epsilon^{ij} C_j \right)\left(i\frac{\delta}{\delta A_i} + \frac{k}{4\delta_\perp}\epsilon^{ik} C_k \right) \\
&\quad -\frac{1}{2}\left(i\frac{\delta}{\delta C_i} - \frac{k}{4\delta_\perp}\epsilon^{ij} A_j \right)\left(i\frac{\delta}{\delta C_i} - \frac{k}{4\delta_\perp}\epsilon^{ik} A_k \right) \\
&\quad +\frac{1}{4}F^{ij}F^{ij} - \frac{1}{4}G^{ij}G^{ij} \Bigg] \Phi_{(0,0)}[A,C] \\
&= \mathcal{E}_{(0,0)} \Phi_{(0,0)}[A,C]
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$$A^i = \pm \frac{\epsilon^{ij}}{k^2} \frac{r_j - \bar{r}_j}{|r - \bar{r}|} \qquad C^i = \pm \frac{\epsilon^{ij}}{k^2} \frac{r_j - \bar{r}_j}{|r - \bar{r}|}$$

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$$\Psi_A[C] = \left(1 - i\frac{k}{4\delta_\perp}\epsilon^{ij} A_i^{\rm ext} C_j + \dots\right)\Phi_0[C]$$

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But that is another story...