Canonical Functional (Holomorphic) Quantization of Pseudo-Photons in Planar Systems

Pedro Castelo Ferreira

CENTRA, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

$\texttt{hep}-\texttt{th}/\texttt{0609239}$	$1.\ \mathrm{Maxwell\ Action}$
$\texttt{hep}-\texttt{th}/\texttt{0703193}$	$2.\ \mathrm{Extended}\ U_e(1)\times U_g(1)\ \mathrm{Electromagnetism}$
$\texttt{hep}-\texttt{th}/\texttt{0703194}$	$3.\ \mathrm{Functional\ Quantization}$

5th September of 2007 - XVI International Fall Workshop on Geometry and Physics

1. Maxwell Action

1. Maxwell Action

Once upon a time

1. Maxwell Action

Once upon a time the Maxwell equations ...

1. Maxwell Action

Once upon a time the Maxwell equations ...

$$
\begin{aligned}
\nabla \cdot \mathbf{E} &= \rho_e \\
\nabla \times \mathbf{B} - \dot{\mathbf{E}} &= \mathbf{j}_e \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{E} + \dot{\mathbf{B}} &= 0
\end{aligned}
$$

1. Maxwell Action

Once upon a time the Maxwell equations ...

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \rho_e \\ \nabla \times \mathbf{B} - \dot{\mathbf{E}} &= \mathbf{j}_e \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} + \dot{\mathbf{B}} &= 0 \end{aligned}$$

... derived phenomenologically and unified in 1861.

1. Maxwell Action

Variational Unification of Electric and Magnetic Interactions is Achieved by Maxwell Action:

1. Maxwell Action

Variational Unification of Electric and Magnetic Interactions is Achieved by Maxwell Action:

$$S_{\mathrm{Maxwell}} = -\int d^4x \left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\nu J_e^\nu\right]$$

1. Maxwell Action

Variational Unification of Electric and Magnetic Interactions is Achieved by Maxwell Action:

$$S_{\mathrm{Maxwell}} = -\int d^4x \left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\nu J_e^\nu\right]$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad , \quad J_e^\nu = (\rho_e, \mathbf{j}_e)$$

1. Maxwell Action

Variational Unification of Electric and Magnetic Interactions is Achieved by Maxwell Action:

$$S_{\mathrm{Maxwell}} = -\int d^4x \left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\nu J_e^\nu\right]$$

Physical Photon: A_μ field

1. Maxwell Action

Variational Unification of Electric and Magnetic Interactions is Achieved by Maxwell Action:

$$S_{\mathrm{Maxwell}} = -\int d^4x \left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\nu J_e^\nu\right]$$

Physical Photon: A_μ field

Equations of Motion: $\dfrac{\delta S}{\delta A_\mu} = 0$

1. Maxwell Action

Variational Unification of Electric and Magnetic Interactions is Achieved by Maxwell Action:

$$S_{\mathrm{Maxwell}} = -\int d^4x \left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\nu J_e^\nu\right]$$

Physical Photon: A_μ field

EOM $\Rightarrow\ \partial_\mu F^{\mu\nu} = J_e^\nu$

1. Maxwell Action

Variational Unification of Electric and Magnetic Interactions is Achieved by Maxwell Action:

$$S_{\mathrm{Maxwell}} = -\int d^4x \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A_\nu J_e^\nu \right]$$

Physical Photon: A_μ field

$$\text{EOM} \;\Rightarrow\; \partial_\mu F^{\mu\nu} = J_e^\nu \;\Leftrightarrow\; \left\{ \begin{array}{rcl} \nabla \cdot \mathbf{E} & = & \rho_e \\ \nabla \times \mathbf{B} - \dot{\mathbf{E}} & = & \mathbf{j}_e \end{array} \right.$$

1. Maxwell Action

Variational Unification of Electric and Magnetic Interactions is Achieved by Maxwell Action:

$$S_{\mathrm{Maxwell}} = -\int d^4x \left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\nu J_e^\nu\right]$$

Physical Photon: A_μ field

EOM $\Rightarrow \partial_\mu F^{\mu\nu} = J_e^\nu \Leftrightarrow \left\{ \begin{array}{rcl} \nabla\cdot\mathbf{E} & = & \rho_e \\ \nabla\times\mathbf{B} - \dot{\mathbf{E}} & = & \mathbf{j}_e \end{array} \right.$

Regularity of Gauge Fields

1. Maxwell Action

Variational Unification of Electric and Magnetic Interactions is Achieved by Maxwell Action:

$$S_{\mathrm{Maxwell}} = -\int d^4x \left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\nu J_e^\nu\right]$$

Physical Photon: A_μ field

EOM $\Rightarrow \partial_\mu F^{\mu\nu} = J_e^\nu \Leftrightarrow \left\{\begin{array}{rcl} \nabla\cdot\mathbf{E} & = & \rho_e \\ \nabla\times\mathbf{B} - \dot{\mathbf{E}} & = & \mathbf{j}_e \end{array}\right.$

Regularity of Gauge Fields $\Rightarrow$ Bianchi Identities

1. Maxwell Action

Variational Unification of Electric and Magnetic Interactions is Achieved by Maxwell Action:

$$S_{\mathrm{Maxwell}} = -\int d^4x \left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\nu J_e^\nu\right]$$

Physical Photon: A_μ field

EOM $\Rightarrow\ \partial_\mu F^{\mu\nu} = J_e^\nu\ \Leftrightarrow \left\{\begin{array}{rcl} \nabla\cdot\mathbf{E} & = & \rho_e \\ \nabla\times\mathbf{B} - \dot{\mathbf{E}} & = & \mathbf{j}_e \end{array}\right.$

BI $\Rightarrow\ \epsilon^{\mu\nu\lambda\rho}\partial_\nu F_{\lambda\rho} = 0$

1. Maxwell Action

Variational Unification of Electric and Magnetic Interactions is Achieved by Maxwell Action:

$$S_{\mathrm{Maxwell}} = -\int d^4x \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A_\nu J_e^\nu\right]$$

Physical Photon: A_μ field

$$\text{EOM} \;\Rightarrow\; \partial_\mu F^{\mu\nu} = J_e^\nu \;\Leftrightarrow\; \left\{ \begin{array}{rcl} \nabla \cdot \mathbf{E} & = & \rho_e \\ \nabla \times \mathbf{B} - \dot{\mathbf{E}} & = & \mathbf{j}_e \end{array} \right.$$

$$\text{BI} \;\Rightarrow\; \epsilon^{\mu\nu\lambda\rho} \partial_\nu F_{\lambda\rho} = 0 \;\Leftrightarrow\; \left\{ \begin{array}{rcl} \nabla \cdot \mathbf{B} & = & 0 \\ \nabla \times \mathbf{E} + \dot{\mathbf{B}} & = & 0 \end{array} \right.$$

1. Maxwell Action

Variational Unification of Electric and Magnetic Interactions is Achieved by Maxwell Action:

$$S_{\mathrm{Maxwell}} = -\int d^4x \left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\nu J_e^\nu\right]$$

Usual definitions: $E^i = F^{0i} \quad , \quad B^i = \frac{1}{2}\epsilon^{ijk}F_{jk}$

$$\text{EOM} \;\Rightarrow\; \partial_\mu F^{\mu\nu} = J_e^\nu \;\Leftrightarrow\; \left\{ \begin{array}{rcl} \nabla\cdot\mathbf{E} & = & \rho_e \\ \nabla\times\mathbf{B} - \dot{\mathbf{E}} & = & \mathbf{j}_e \end{array} \right.$$

$$\text{BI} \;\Rightarrow\; \epsilon^{\mu\nu\lambda\rho}\partial_\nu F_{\lambda\rho} = 0 \;\Leftrightarrow\; \left\{ \begin{array}{rcl} \nabla\cdot\mathbf{B} & = & 0 \\ \nabla\times\mathbf{E} + \dot{\mathbf{B}} & = & 0 \end{array} \right.$$

1. Maxwell Action

- Sucessfull in most fields of physics

1. Maxwell Action

- Sucessfull in most fields of physics, both at classical and quantum level

1. Maxwell Action

- Sucessfull in most fields of physics, both at classical and quantum level

- However does not reproduce the Bianchi Identities which are imposed externally

1. Maxwell Action

- Sucessfull in most fields of physics, both at classical and quantum level

- However does not reproduce the Bianchi Identities which are imposed externally

$\Rightarrow$ Hence fails to describe the full Maxwell Equations

at variational level

1. Maxwell Action

This is a relevant issue in the particular cases:

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ

$$\text{BI} \;\Rightarrow\; \epsilon^{\mu\nu\lambda\rho}\partial_\nu F_{\lambda\rho} = J_g^\mu$$

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ

$$\text{BI} \;\Rightarrow\; \epsilon^{\mu\nu\lambda\rho}\partial_\nu F_{\lambda\rho} = J_g^\mu \;\Leftrightarrow\; \left\{ \begin{array}{rcl} \nabla\cdot\mathbf{B} & = & \rho_g \\ \nabla\times\mathbf{E}+\dot{\mathbf{B}} & = & \mathbf{j}_g^\mu \end{array} \right.$$

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ

$$\text{BI} \;\Rightarrow\; \epsilon^{\mu\nu\lambda\rho}\partial_\nu F_{\lambda\rho} = J_g^\mu \;\Leftrightarrow\; \left\{ \begin{array}{rcl} \nabla \cdot \mathbf{B} & = & \rho_g \\ \nabla \times \mathbf{E} + \dot{\mathbf{B}} & = & \mathbf{j}_g^\mu \end{array} \right.$$

– Not deduceable from from the Maxwell action

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ

$$\text{BI} \;\Rightarrow\; \epsilon^{\mu\nu\lambda\rho}\partial_\nu F_{\lambda\rho} = J_g^\mu \;\Leftrightarrow\; \begin{cases} \nabla \cdot \mathbf{B} &= \rho_g \\ \nabla \times \mathbf{E} + \dot{\mathbf{B}} &= \mathbf{j}_g^\mu \end{cases}$$

– Not deduceable from from the Maxwell action

– Implies extended singularities

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ

$$\text{BI} \;\Rightarrow\; \epsilon^{\mu\nu\lambda\rho}\partial_\nu F_{\lambda\rho} = J_g^\mu \;\Leftrightarrow\; \left\{ \begin{array}{rcl} \nabla\cdot\mathbf{B} & = & \rho_g \\ \nabla\times\mathbf{E}+\dot{\mathbf{B}} & = & \mathbf{j}_g^\mu \end{array} \right.$$

– Not deduceable from from the Maxwell action

– Implies extended singularities

in principle non-physical

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J^μ_g

$$\text{BI} \;\Rightarrow\; \epsilon^{\mu\nu\lambda\rho}\partial_\nu F_{\lambda\rho} = J^\mu_g \;\Leftrightarrow\; \left\{ \begin{array}{rcl} \nabla\cdot\mathbf{B} & = & \rho_g \\ \nabla\times\mathbf{E}+\dot{\mathbf{B}} & = & \mathbf{j}^\mu_g \end{array} \right.$$

– Not deduceable from from the Maxwell action

– Implies extended singularities

either the Dirac string of Wu-Yang fiber-bundle

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ
- non-regular external electromagnetic fields

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ
- non-regular external electromagnetic fields

$$\dot{\mathbf{B}} \neq 0$$

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ
- non-regular external electromagnetic fields

$$\begin{aligned} \dot{\mathbf{B}} &\neq 0 \\ \nabla \times \mathbf{E} &\neq 0 \end{aligned}$$

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ
- non-regular external electromagnetic fields

$$\left.\begin{aligned} \dot{\mathbf{B}} &\neq 0 \\ \nabla \times \mathbf{E} &\neq 0 \end{aligned}\right\} \Rightarrow \nabla \times \mathbf{E} + \dot{\mathbf{B}} \neq \mathbf{0}$$

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ
- non-regular external electromagnetic fields

$$\left.\begin{array}{rcl} \dot{\mathbf{B}} & \neq & 0 \\ \nabla \times \mathbf{E} & \neq & 0 \end{array}\right\} \Rightarrow \nabla \times \mathbf{E} + \dot{\mathbf{B}} \neq \mathbf{0}$$

– violates Bianchi Identities

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ

- non-regular external electromagnetic fields

$$\left.\begin{array}{rcl} \dot{\mathbf{B}} & \neq & 0 \\ \\ \nabla \times \mathbf{E} & \neq & 0 \end{array}\right\} \Rightarrow \nabla \times \mathbf{E} + \dot{\mathbf{B}} \neq \mathbf{0}$$

  – violates Bianchi Identities
  – is not deduceable from from the Maxwell action

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ
- non-regular external electromagnetic fields

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ
- non-regular external electromagnetic fields

– although magnetic monopoles are the only justification for quantization of electric charge have so far not been detected

1. Maxwell Action

This is a relevant issue in the particular cases:

- presence of magnetic currents J_g^μ
- non-regular external electromagnetic fields

– although magnetic monopoles are the only justification for quantization of electric charge have so far not been detected

– however non-regular external electromagnetic fields are common in many physical systems and there is plenty of experimental evidence of violation of the Bianchi Identities

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism

A possible solution is to consider an extend gauge group

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism

A possible solution is to consider an extend gauge group

$$U_e(1) \times U_g(1)$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism

A possible solution is to consider an extend gauge group

$$U_e(1) \times U_g(1) : \quad \begin{cases} \mathrm{Photon} : & A_\mu \\ & \\ \mathrm{Pseudo-Photon} : & C_\mu \end{cases}$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism

A possible solution is to consider an extend gauge group

$$U_e(1) \times U_g(1): \quad \begin{cases} \mathrm{Photon}: & A_\mu \\ \\ \mathrm{Pseudo-Photon}: & C_\mu \end{cases}$$

$$S = \frac{1}{4} \int d^4x \left[-F_{\mu\nu}F^{\mu\nu} \ + G_{\mu\nu}G^{\mu\nu} \ + \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}G_{\lambda\rho} \right]$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism

A possible solution is to consider an extend gauge group

$$U_e(1) \times U_g(1): \quad \begin{cases} \mathrm{Photon}: & A_\mu \\ \\ \mathrm{Pseudo-Photon}: & C_\mu \end{cases}$$

$$S = \frac{1}{4} \int d^4x \left[-F_{\mu\nu}F^{\mu\nu} \ + G_{\mu\nu}G^{\mu\nu} \ + \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}G_{\lambda\rho} \right]$$

Originaly motivated by the inclusion of magnetic monopoles

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism

A possible solution is to consider an extend gauge group

$$U_e(1) \times U_g(1) : \quad \begin{cases} \mathrm{Photon:} & A_\mu \\ & \\ \mathrm{Pseudo-Photon:} & C_\mu \end{cases}$$

$$S = \frac{1}{4} \int d^4x \left[-F_{\mu\nu}F^{\mu\nu} \ + G_{\mu\nu}G^{\mu\nu} \ + \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}G_{\lambda\rho} \right]$$

Originaly motivated by the inclusion of magnetic monopoles implies two distinct coupling constants e and g

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism

A possible solution is to consider an extend gauge group

$$U_e(1) \times U_g(1) : \quad \begin{cases} \text{Photon :} & A_\mu \\ & \\ \text{Pseudo} - \text{Photon :} & C_\mu \end{cases}$$

$$S = \frac{1}{4} \int d^4x \left[-\frac{1}{e^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{g^2} G_{\mu\nu} G^{\mu\nu} + \frac{1}{e\,g} \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} G_{\lambda\rho} \right]$$

Originaly motivated by the inclusion of magnetic monopoles implies two distinct coupling constants e and g

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism

A possible solution is to consider an extend gauge group

$$U_e(1) \times U_g(1) : \quad \begin{cases} \mathrm{Photon:} & A_\mu \\ & \\ \mathrm{Pseudo-Photon:} & C_\mu \end{cases}$$

$$S = \frac{1}{4}\int d^4x \left[-\frac{1}{e^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{g^2}G_{\mu\nu}G^{\mu\nu} + \frac{1}{e\,g}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}G_{\lambda\rho}\right]$$

Obeying Dirac's Quantization Condition

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism

A possible solution is to consider an extend gauge group

$$U_e(1) \times U_g(1): \quad \begin{cases} \mathrm{Photon}: & A_\mu \\ \\ \mathrm{Pseudo-Photon}: & C_\mu \end{cases}$$

$$S = \frac{1}{4} \int d^4x \left[-\frac{1}{e^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{g^2} G_{\mu\nu} G^{\mu\nu} + \frac{1}{e\,g} \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} G_{\lambda\rho} \right]$$

Obeying Dirac's Quantization Condition

$$eg = 2\pi\,\hbar\,n$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Monopoles

Inclusion of currents J_e^μ and J_g^μ

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Monopoles

Inclusion of currents J_e^μ and J_g^μ

$$S = \frac{1}{4}\int d^4x \left[-F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} + \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}G_{\lambda\rho}\right]$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Monopoles

Inclusion of currents J_e^μ and J_g^μ

$$S = \frac{1}{4}\int d^4x \left[-F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} + \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}G_{\lambda\rho}\right]$$

$$+ \int d^4x \left[e\left(A_\mu - \tilde{C}_\mu\right)J_e^\mu\right]$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Monopoles

Inclusion of currents J_e^μ and J_g^μ

$$
\begin{aligned}
S &= \frac{1}{4}\int d^4x \left[-F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} + \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}G_{\lambda\rho}\right] \\
&\quad + \int d^4x \left[e\left(A_\mu - \tilde{C}_\mu\right)J_e^\mu + g\left(A_\mu - \tilde{C}_\mu\right)J_g^\mu\right]
\end{aligned}
$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Monopoles

Inclusion of currents J_e^μ and J_g^μ

$$
\begin{aligned}
S &= \frac{1}{4}\int d^4x \left[-F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} + \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}G_{\lambda\rho}\right] \\
&\quad + \int d^4x \left[e\left(A_\mu - \tilde{C}_\mu\right)J_e^\mu + g\left(A_\mu - \tilde{C}_\mu\right)J_g^\mu\right]
\end{aligned}
$$

$$
\tilde{A}: \ \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}
$$

$$
\tilde{C}: \ \tilde{G}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho}
$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Monopoles

Inclusion of currents J_e^μ and J_g^μ

$$
\begin{aligned}
S &= \frac{1}{4}\int d^4x \left[-F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} + \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}G_{\lambda\rho}\right] \\
&+ \int d^4x \left[e\left(A_\mu - \tilde{C}_\mu\right)J_e^\mu + g\left(A_\mu - \tilde{C}_\mu\right)J_g^\mu\right]
\end{aligned}
$$

EOM A

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Monopoles

Inclusion of currents J_e^μ and J_g^μ

$$
\begin{aligned}
S &= \frac{1}{4}\int d^4x\left[-F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} + \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}G_{\lambda\rho}\right] \\
&\quad + \int d^4x\left[e\left(A_\mu - \tilde{C}_\mu\right)J_e^\mu + g\left(A_\mu - \tilde{C}_\mu\right)J_g^\mu\right]
\end{aligned}
$$

EOM A : $\partial_\mu\left(F^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho}\right) = J_e^\nu$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Monopoles

Inclusion of currents J_e^μ and J_g^μ

$$S = \frac{1}{4}\int d^4x \left[-F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} + \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}G_{\lambda\rho}\right]$$
$$+ \int d^4x \left[e\left(A_\mu - \tilde{C}_\mu\right)J_e^\mu + g\left(A_\mu - \tilde{C}_\mu\right)J_g^\mu\right]$$

EOM A : $\partial_\mu\left(F^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho}\right) = J_e^\nu$

EOM C

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Monopoles

Inclusion of currents J_e^μ and J_g^μ

$$S = \frac{1}{4}\int d^4x \left[-F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} + \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}G_{\lambda\rho}\right]$$

$$+ \int d^4x \left[e\left(A_\mu - \tilde{C}_\mu\right)J_e^\mu + g\left(A_\mu - \tilde{C}_\mu\right)J_g^\mu\right]$$

EOM A : $\partial_\mu\left(F^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho}\right) = J_e^\nu$

EOM C : $\partial_\mu\left(G^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}\right) = J_g^\nu$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Monopoles

Inclusion of currents J_e^μ and J_g^μ

$$S = \frac{1}{4}\int d^4x \left[-F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} + \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}G_{\lambda\rho}\right]$$

$$+ \int d^4x \left[e\left(A_\mu - \tilde{C}_\mu\right)J_e^\mu + g\left(A_\mu - \tilde{C}_\mu\right)J_g^\mu\right]$$

EOM A : $\partial_\mu\left(F^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho}\right) = J_e^\nu$

EOM C : $\partial_\mu\left(G^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}\right) = J_g^\nu$

$$E^i = F^{0i} - \frac{1}{2}\epsilon^{0ijk}G_{jk} \quad , \quad B^i = G^{0i} + \frac{1}{2}\epsilon^{0ijk}F_{jk}$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Monopoles

Inclusion of currents J_e^μ and J_g^μ

$$S = \frac{1}{4}\int d^4x \left[-F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} + \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}G_{\lambda\rho}\right]$$

$$+ \int d^4x \left[e\left(A_\mu - \tilde{C}_\mu\right)J_e^\mu + g\left(A_\mu - \tilde{C}_\mu\right)J_g^\mu\right]$$

EOM A : $\partial_\mu\left(F^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho}\right) = J_e^\nu \Leftrightarrow \left\{\begin{array}{rcl}\nabla\cdot\mathbf{E} & = & \rho_e \\ \nabla\times\mathbf{B} - \dot{\mathbf{E}} & = & \mathbf{j}_e\end{array}\right.$

EOM C : $\partial_\mu\left(G^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}\right) = J_g^\nu$

$$E^i = F^{0i} - \frac{1}{2}\epsilon^{0ijk}G_{jk} \quad , \quad B^i = G^{0i} + \frac{1}{2}\epsilon^{0ijk}F_{jk}$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Monopoles

Inclusion of currents J_e^μ and J_g^μ

$$S = \frac{1}{4}\int d^4x\left[-F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu} + \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}G_{\lambda\rho}\right]$$

$$+\int d^4x\left[e\left(A_\mu - \tilde{C}_\mu\right)J_e^\mu + g\left(A_\mu - \tilde{C}_\mu\right)J_g^\mu\right]$$

EOM A : $\partial_\mu\left(F^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho}\right) = J_e^\nu \;\Leftrightarrow\; \left\{\begin{array}{rcl}\nabla\cdot\mathbf{E} & = & \rho_e \\ \nabla\times\mathbf{B} - \dot{\mathbf{E}} & = & \mathbf{j}_e\end{array}\right.$

EOM C : $\partial_\mu\left(G^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}\right) = J_g^\nu \;\Leftrightarrow\; \left\{\begin{array}{rcl}\nabla\cdot\mathbf{B} & = & \rho_g \\ \nabla\times\mathbf{E} + \dot{\mathbf{B}} & = & \mathbf{j}_g\end{array}\right.$

$$E^i = F^{0i} - \frac{1}{2}\epsilon^{0ijk}G_{jk} \quad , \quad B^i = G^{0i} + \frac{1}{2}\epsilon^{0ijk}F_{jk}$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

Let us consider a decomposition $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$ into:

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

Let us consider a decomposition $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$ into:

- External Fields

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

Let us consider a decomposition $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$ into:

- External Fields

- Internal Fields

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

Let us consider a decomposition $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$ into:

- External Fields: $\bar{F}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$
- Internal Fields

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

Let us consider a decomposition $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$ into:

- External Fields: $\bar{F}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$
- Internal Fields : $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \quad , \quad G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

Let us consider a decomposition $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$ into:

- External Fields: $\bar{F}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$
- Internal Fields : $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \quad , \quad G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$

Such that the action terms decompose as :

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

Let us consider a decomposition $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$ into:

- External Fields: $\bar{F}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$
- Internal Fields : $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \quad , \quad G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$

Such that the action terms decompose as :

$$F_{\mu\nu}F^{\mu\nu} = f_{\mu\nu}f^{\mu\nu} + \bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + 2\bar{F}_{\mu\nu}f^{\mu\nu}$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

Let us consider a decomposition $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$ into:

- External Fields: $\bar{F}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$
- Internal Fields : $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \quad , \quad G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$

Such that the action terms decompose as :

$$F_{\mu\nu} F^{\mu\nu} = f_{\mu\nu} f^{\mu\nu} + \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} + 2\bar{F}_{\mu\nu} f^{\mu\nu}$$

$$\epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} G_{\lambda\rho} = +\epsilon^{\mu\nu\delta\rho} f_{\mu\nu} G_{\delta\rho} + \epsilon^{\mu\nu\delta\rho} \bar{F}_{\mu\nu} G_{\delta\rho}$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

Let us consider a decomposition $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$ into:

- External Fields: $\bar{F}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$
- Internal Fields : $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \quad , \quad G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$

And the action

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

Let us consider a decomposition $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$ into:

- External Fields: $\bar{F}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$
- Internal Fields : $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \quad , \quad G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$

And the action

$$S = \frac{1}{4} \int dx^4 \left[-F_{\mu\nu} F^{\mu\nu} \right.$$
$$\left. + G_{\mu\nu} G^{\mu\nu} + \epsilon^{\mu\nu\delta\rho} F_{\mu\nu} G_{\delta\rho} \right]$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

Let us consider a decomposition $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$ into:

- External Fields: $\bar{F}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$
- Internal Fields : $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \quad , \quad G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$

And the action decomposes accordingly :

$$S = \frac{1}{4} \int dx^4 \left[-F_{\mu\nu} F^{\mu\nu} \right.$$

$$\left. + G_{\mu\nu} G^{\mu\nu} + \epsilon^{\mu\nu\delta\rho} F_{\mu\nu} G_{\delta\rho} \right]$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

Let us consider a decomposition $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$ into:

- External Fields: $\bar{F}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$
- Internal Fields : $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \quad , \quad G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$

And the action decomposes accordingly :

$$S = \frac{1}{4} \int dx^4 \left[-f_{\mu\nu} f^{\mu\nu} - \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} - 2\bar{F}_{\mu\nu} f^{\mu\nu} \right.$$

$$\left. + G_{\mu\nu} G^{\mu\nu} + \epsilon^{\mu\nu\delta\rho} F_{\mu\nu} G_{\delta\rho} \right]$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

Let us consider a decomposition $F_{\mu\nu} = \bar{F}_{\mu\nu} + f_{\mu\nu}$ into:

- External Fields: $\bar{F}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$
- Internal Fields : $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \quad , \quad G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$

And the action decomposes accordingly :

$$S = \frac{1}{4}\int dx^4 \left[-f_{\mu\nu}f^{\mu\nu} - \bar{F}_{\mu\nu}\bar{F}^{\mu\nu} - 2\bar{F}_{\mu\nu}f^{\mu\nu}\right.$$

$$\left. + G_{\mu\nu}G^{\mu\nu} + \epsilon^{\mu\nu\delta\rho} f_{\mu\nu}G_{\delta\rho} + \epsilon^{\mu\nu\delta\rho}\bar{F}_{\mu\nu}G_{\delta\rho}\right]$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

The equations of motion are obtained as previously:

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

The equations of motion are obtained as previously:

EOM a : $\partial_\mu \left(f^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} G_{\lambda\rho} \right) + \partial_\mu \bar{F}_{\mu\nu} = 0$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

The equations of motion are obtained as previously:

$$\text{EOM } a \ : \partial_\mu \left(f^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} G_{\lambda\rho} \right) + \partial_\mu \bar{F}_{\mu\nu} = 0$$

$$\text{EOM } C : \partial_\mu \left(G^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} f_{\lambda\rho} \right) + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} \partial_\nu \bar{F}_{\lambda\rho} = 0$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

The equations of motion are obtained as previously:

$$\text{EOM } a \ : \partial_\mu \left(f^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} G_{\lambda\rho} \right) + \partial_\mu \bar{F}_{\mu\nu} = 0$$

$$\text{EOM } C : \partial_\mu \left(G^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} f_{\lambda\rho} \right) + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} \partial_\nu \bar{F}_{\lambda\rho} = 0$$

Electromagnetic field definitions:

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

The equations of motion are obtained as previously:

$$\text{EOM } a \ : \partial_\mu \left(f^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} G_{\lambda\rho} \right) + \partial_\mu \bar{F}_{\mu\nu} = 0$$

$$\text{EOM } C : \partial_\mu \left(G^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} f_{\lambda\rho} \right) + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} \partial_\nu \bar{F}_{\lambda\rho} = 0$$

Electromagnetic field definitions:

$$E^{\mathrm{ext}\, i} = \bar{F}^{0i} \qquad , \qquad B^{\mathrm{ext}\, i} = \frac{1}{2}\epsilon^{0ijk} \bar{F}_{jk}$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

The equations of motion are obtained as previously:

$$\text{EOM } a \ : \partial_\mu \left(f^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} G_{\lambda\rho} \right) + \partial_\mu \bar{F}_{\mu\nu} = 0$$

$$\text{EOM } C : \partial_\mu \left(G^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} f_{\lambda\rho} \right) + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} \partial_\nu \bar{F}_{\lambda\rho} = 0$$

Electromagnetic field definitions:

$$E^{\text{ext}\,i} = \bar{F}^{0i} \quad , \quad B^{\text{ext}\,i} = \frac{1}{2}\epsilon^{0ijk}\bar{F}_{jk}$$

$$E^i = f^{0i} - \frac{1}{2}\epsilon^{0ijk} G_{jk} \quad , \quad B^i = G^{0i} + \frac{1}{2}\epsilon^{0ijk} f_{jk}$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

The equations of motion are obtained as previously:

$$\text{EOM } a \ : \left\{ \begin{array}{rcl} \nabla \cdot \mathbf{E} + \nabla \cdot \mathbf{E}^{\text{ext}} & = & 0 \\ \nabla \times \mathbf{B} - \dot{\mathbf{E}} + \nabla \times \mathbf{B}^{\text{ext}} - \dot{\mathbf{E}}^{\text{ext}} & = & 0 \end{array} \right.$$

$$\text{EOM } C : \partial_\mu \left(G^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} f_{\lambda\rho} \right) + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}\partial_\nu \bar{F}_{\lambda\rho} = 0$$

Electromagnetic field definitions:

$$E^{\text{ext}\, i} = \bar{F}^{0i} \quad , \quad B^{\text{ext}\, i} = \frac{1}{2}\epsilon^{0ijk}\bar{F}_{jk}$$

$$E^i = f^{0i} - \frac{1}{2}\epsilon^{0ijk} G_{jk} \quad , \quad B^i = G^{0i} + \frac{1}{2}\epsilon^{0ijk} f_{jk}$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – External Fields

The equations of motion are obtained as previously:

$$\text{EOM } a \; : \left\{ \begin{array}{rcl} \nabla \cdot \mathbf{E} + \nabla \cdot \mathbf{E}^{\mathrm{ext}} & = & 0 \\ \nabla \times \mathbf{B} - \dot{\mathbf{E}} + \nabla \times \mathbf{B}^{\mathrm{ext}} - \dot{\mathbf{E}}^{\mathrm{ext}} & = & 0 \end{array} \right.$$

$$\text{EOM } C : \left\{ \begin{array}{rcl} \nabla \cdot \mathbf{B} + \nabla \cdot \mathbf{B}^{\mathrm{ext}} & = & 0 \\ \nabla \times \mathbf{E} + \dot{\mathbf{B}} + \nabla \times \mathbf{E}^{\mathrm{ext}} + \dot{\mathbf{B}}^{\mathrm{ext}} & = & 0 \end{array} \right.$$

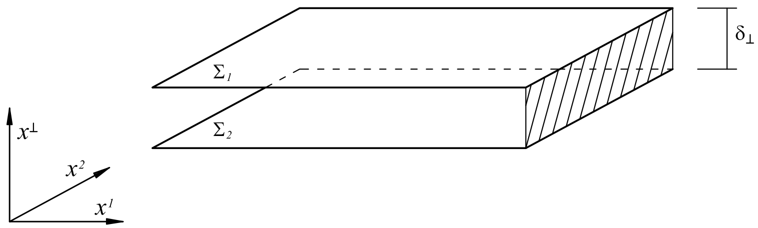
Electromagnetic field definitions:

$$E^{\mathrm{ext}\, i} = \bar{F}^{0i} \quad , \quad B^{\mathrm{ext}\, i} = \frac{1}{2}\epsilon^{0ijk}\bar{F}_{jk}$$

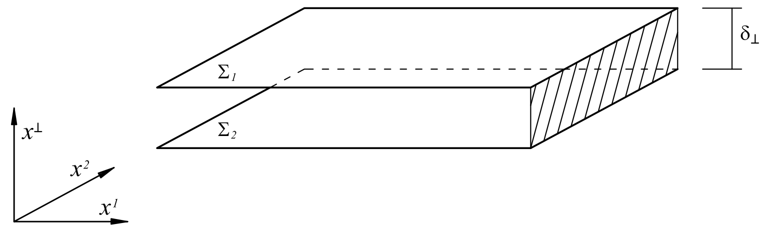
$$E^i = f^{0i} - \frac{1}{2}\epsilon^{0ijk}G_{jk} \quad , \quad B^i = G^{0i} + \frac{1}{2}\epsilon^{0ijk}f_{jk}$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction

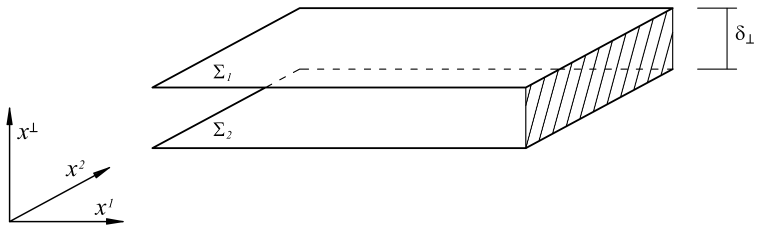


2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



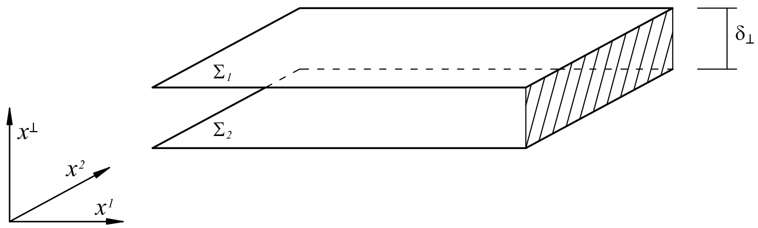
$$I, J = \mu, \perp \quad ; \quad \mu = 0, i, j \quad ; \quad i, j = 1, 2$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



Assumptions:

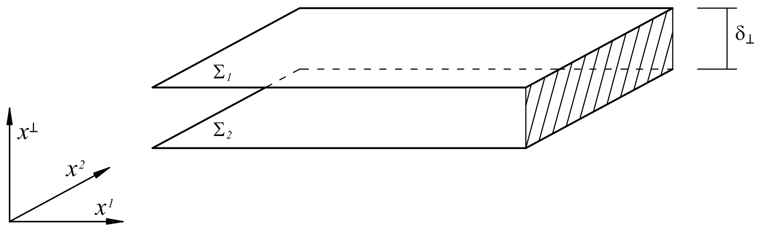
2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



Assumptions:

Field localization: $x^\perp \in [-\delta_\perp/2, +\delta_\perp/2]$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction

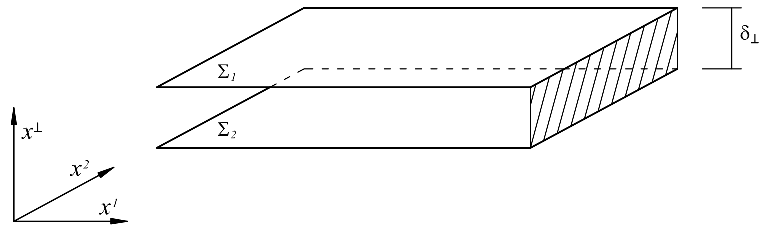


Assumptions:

Field localization: $x^{\perp} \in [-\delta_{\perp}/2, +\delta_{\perp}/2]$

Gauge partially Fixed: $\partial_{\perp}\Lambda = 0$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



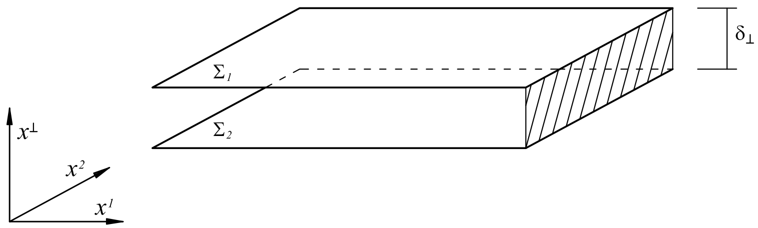
Assumptions:

Field localization: $x^\perp \in [-\delta_\perp/2, +\delta_\perp/2]$

Gauge partially Fixed: $\partial_\perp \Lambda = 0$

Neumann b.c. for fields: $A_\perp = C_\perp = 0$

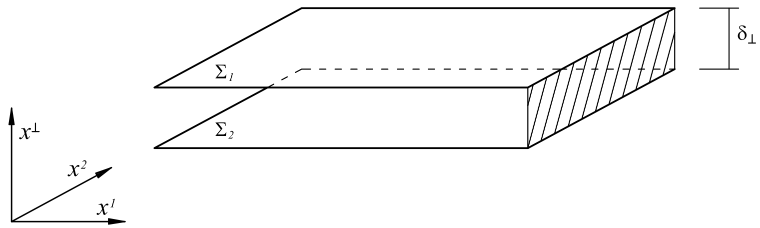
2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



Assumptions:

- Field localization: $x^\perp \in [-\delta_\perp/2, +\delta_\perp/2]$
- Gauge partially Fixed: $\partial_\perp \Lambda = 0$
- Neumann b.c. for fields: $A_\perp = C_\perp = 0$
- Constant orthogonal fields: $\partial_\perp A_\mu = \partial_\perp C_\mu = 0$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



Assumptions:

Field localization: $x^\perp \in [-\delta_\perp/2, +\delta_\perp/2]$

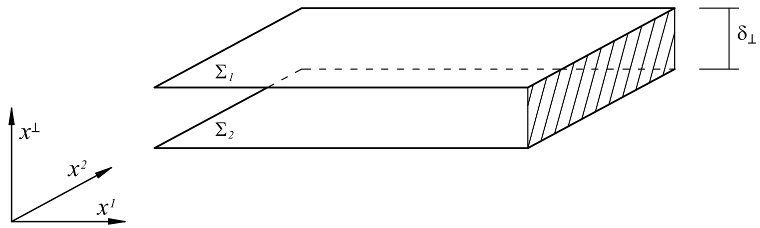
Gauge partially Fixed: $\partial_\perp \Lambda = 0$

Neumann b.c. for fields: $A_\perp = C_\perp = 0$

Constant orthogonal fields: $\partial_\perp A_\mu = \partial_\perp C_\mu = 0$

Regular fields

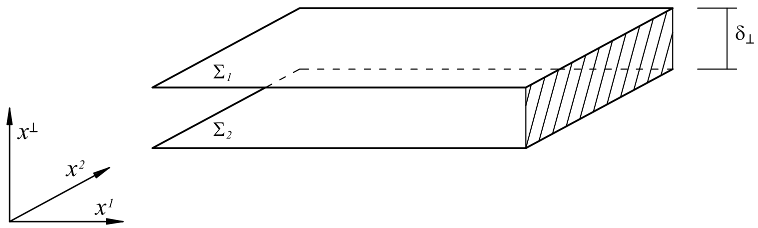
2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



Assumptions:

- Field localization: $x^\perp \in [-\delta_\perp/2, +\delta_\perp/2]$
- Gauge partially Fixed: $\partial_\perp \Lambda = 0$
- Neumann b.c. for fields: $A_\perp = C_\perp = 0$
- Constant orthogonal fields: $\partial_\perp A_\mu = \partial_\perp C_\mu = 0$
- Regular fields
- Identification of boundaries: $\Sigma(x^\perp = 0) \cong \Sigma_1 \cong \Sigma_2$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



Assumptions:

Field localization: $x^{\perp} \in [-\delta_{\perp}/2, +\delta_{\perp}/2]$

Gauge partially Fixed: $\partial_{\perp}\Lambda = 0$

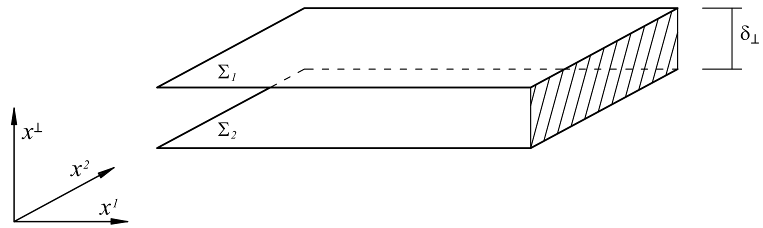
Neumann b.c. for fields: $A_{\perp} = C_{\perp} = 0$

Constant orthogonal fields: $\partial_{\perp}A_{\mu} = \partial_{\perp}C_{\mu} = 0$

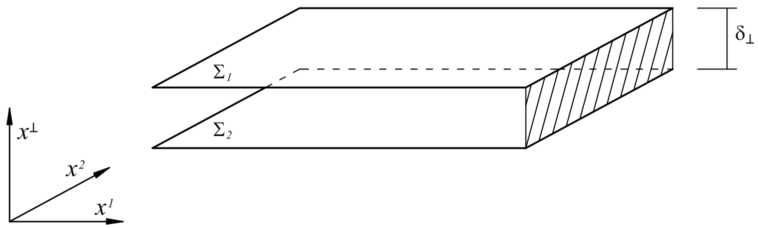
Regular fields

Id. of bound.: $\displaystyle\int_{\Sigma_1 - \Sigma_2} \epsilon^{\mu\nu\lambda} A_{\mu}\partial_{\nu}C_{\lambda} \equiv \frac{k}{2} \int_{\Sigma} \epsilon^{\mu\nu\lambda} A_{\mu}\partial_{\nu}C_{\lambda}$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction

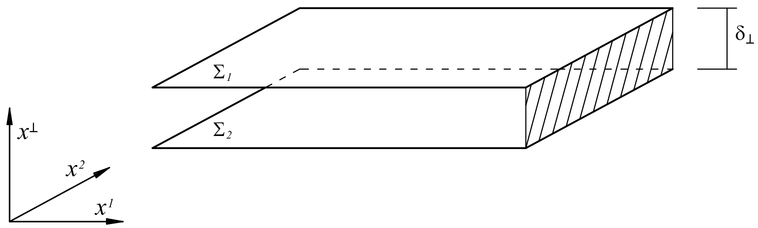


2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



$$\mathcal{L}_4 = -\frac{1}{4} F_{IJ} F^{IJ} + \frac{1}{4} G_{IJ} G^{IJ} + \frac{1}{4} \epsilon^{IJKL} F_{IJ} G_{KL} + A_I \, J_e^I$$

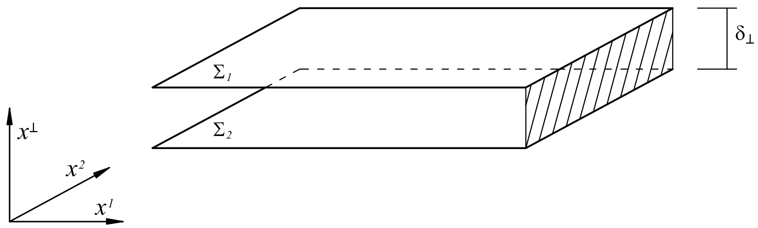
2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



$$\mathcal{L}_4 = -\frac{1}{4}F_{IJ}F^{IJ} + \frac{1}{4}G_{IJ}G^{IJ} + \frac{1}{4}\epsilon^{IJKL}F_{IJ}G_{KL} + A_I\, J_e^I$$

$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction

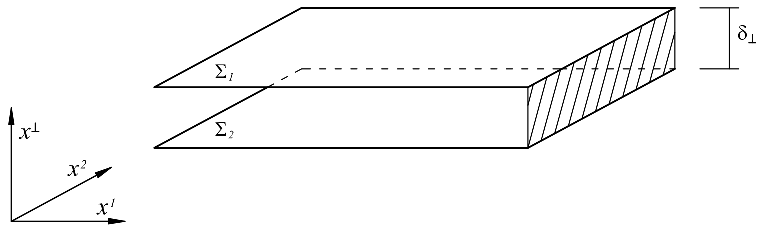


$$\mathcal{L}_4 = -\frac{1}{4}F_{IJ}F^{IJ} + \frac{1}{4}G_{IJ}G^{IJ} + \frac{1}{4}\epsilon^{IJKL}F_{IJ}G_{KL} + A_I\, J_e^I$$

$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$

$$+\epsilon^{\perp\mu\nu\lambda}\partial_\perp\left(A_\mu G_{\nu\lambda}\right) + \epsilon^{\mu\nu\perp\lambda}\partial_\perp\left(\partial_\nu A_\mu C_\lambda\right) + A_\mu\, J_e^\mu$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



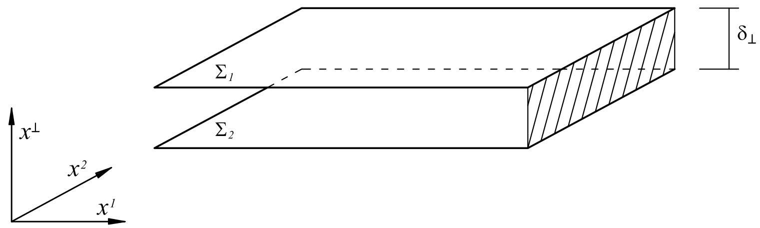
$$\mathcal{L}_4 = -\frac{1}{4} F_{IJ} F^{IJ} + \frac{1}{4} G_{IJ} G^{IJ} + \frac{1}{4} \epsilon^{IJKL} F_{IJ} G_{KL} + A_I \, J_e^I$$

$$= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

$$+ \epsilon^{\perp\mu\nu\lambda} \partial_\perp \left(A_\mu G_{\nu\lambda} \right) + \epsilon^{\mu\nu\perp\lambda} \partial_\perp \left(\partial_\nu A_\mu C_\lambda \right) + A_\mu \, J_e^\mu$$

$$S = \int dx^3 \int dx^\perp \mathcal{L}_4$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



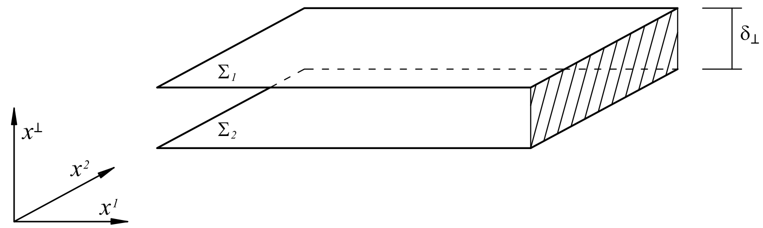
$$\mathcal{L}_4 = -\frac{1}{4}F_{IJ}F^{IJ} + \frac{1}{4}G_{IJ}G^{IJ} + \frac{1}{4}\epsilon^{IJKL}F_{IJ}G_{KL} + A_I\,J_e^I$$

$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$

$$+\epsilon^{\perp\mu\nu\lambda}\partial_\perp\left(A_\mu G_{\nu\lambda}\right) + \epsilon^{\mu\nu\perp\lambda}\partial_\perp\left(\partial_\nu A_\mu C_\lambda\right) + A_\mu\,J_e^\mu$$

$$S = \int dx^3 \int dx^\perp \mathcal{L}_4 = \int dx^3 \mathcal{L}_3$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



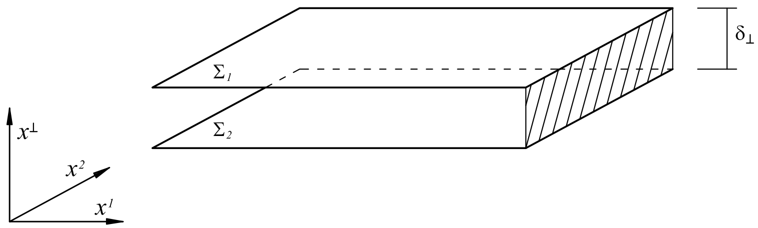
$$\mathcal{L}_4 = -\frac{1}{4}F_{IJ}F^{IJ} + \frac{1}{4}G_{IJ}G^{IJ} + \frac{1}{4}\epsilon^{IJKL}F_{IJ}G_{KL} + A_I\, J_e^I$$

$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$

$$+\epsilon^{\perp\mu\nu\lambda}\partial_\perp\left(A_\mu G_{\nu\lambda}\right) + \epsilon^{\mu\nu\perp\lambda}\partial_\perp\left(\partial_\nu A_\mu C_\lambda\right) + A_\mu\, J_e^\mu$$

$$\mathcal{L}_3 = \int dx^\perp \mathcal{L}_4$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



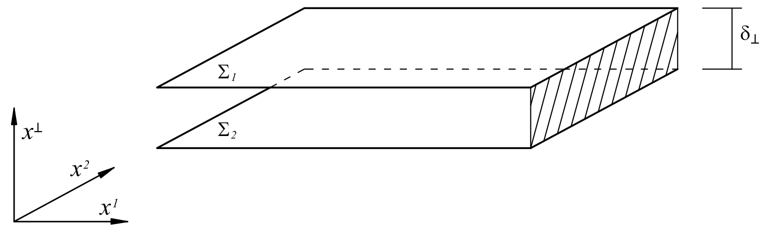
$$\mathcal{L}_4 = -\frac{1}{4}F_{IJ}F^{IJ} + \frac{1}{4}G_{IJ}G^{IJ} + \frac{1}{4}\epsilon^{IJKL}F_{IJ}G_{KL} + A_I\, J_e^I$$

$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$

$$+\epsilon^{\perp\mu\nu\lambda}\partial_\perp\left(A_\mu G_{\nu\lambda}\right) + \epsilon^{\mu\nu\perp\lambda}\partial_\perp\left(\partial_\nu A_\mu C_\lambda\right) + A_\mu\, J_e^\mu$$

$$\mathcal{L}_3 = -\frac{\delta_\perp}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\delta_\perp}{4}G_{\mu\nu}G^{\mu\nu} + \delta_\perp A_\mu J_e^\mu$$

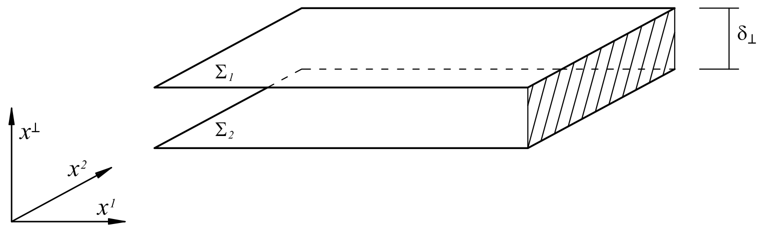
2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



$$\mathcal{L}_4 = -\frac{1}{4}F_{IJ}F^{IJ} + \frac{1}{4}G_{IJ}G^{IJ} + \frac{1}{4}\epsilon^{IJKL}F_{IJ}G_{KL} + A_I\, J_e^I$$
$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$
$$+\epsilon^{\perp\mu\nu\lambda}\partial_\perp\left(A_\mu G_{\nu\lambda}\right) + \epsilon^{\mu\nu\perp\lambda}\partial_\perp\left(\partial_\nu A_\mu C_\lambda\right) + A_\mu\, J_e^\mu$$

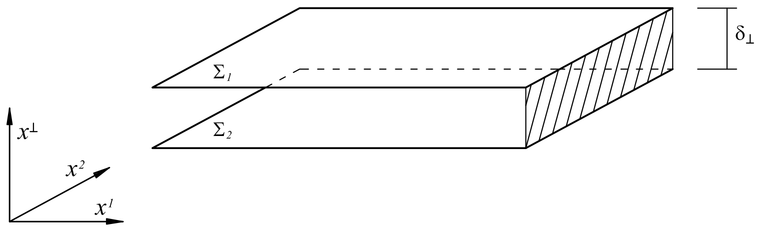
$$\mathcal{L}_3 = -\frac{\delta_\perp}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\delta_\perp}{4}G_{\mu\nu}G^{\mu\nu} + \delta_\perp A_\mu J_e^\mu$$
$$+\frac{k}{8}\epsilon^{\mu\nu\lambda}A_\mu G_{\nu\lambda} + \frac{k}{8}\epsilon^{\mu\nu\lambda}C_\mu F_{\nu\lambda}$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



$$\mathcal{L}_3 = -\frac{\delta_\perp}{4} F_{\mu\nu}F^{\mu\nu} + \frac{\delta_\perp}{4} G_{\mu\nu}G^{\mu\nu} + \frac{k}{8}\epsilon^{\mu\nu\lambda}A_\mu G_{\nu\lambda} + \frac{k}{8}\epsilon^{\mu\nu\lambda}C_\mu F_{\nu\lambda}$$

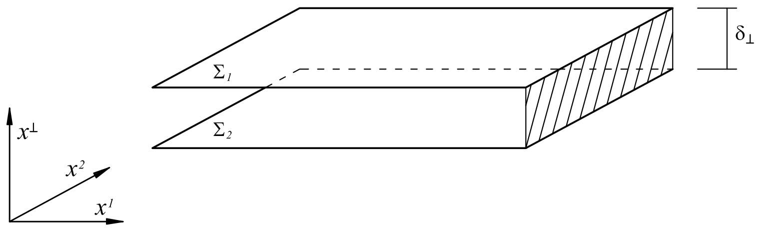
2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



$$\mathcal{L}_3 = -\frac{\delta_\perp}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\delta_\perp}{4}G_{\mu\nu}G^{\mu\nu} + \frac{k}{8}\epsilon^{\mu\nu\lambda}A_\mu G_{\nu\lambda} + \frac{k}{8}\epsilon^{\mu\nu\lambda}C_\mu F_{\nu\lambda}$$

Electric Fields:

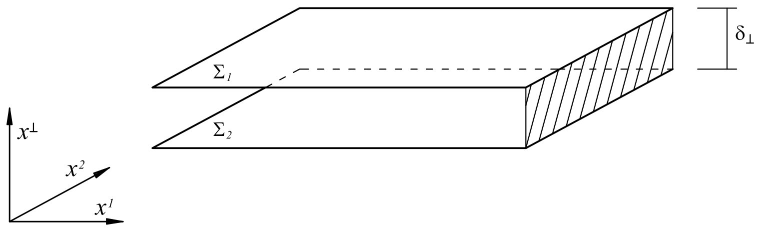
2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



$$\mathcal{L}_3 = -\frac{\delta_\perp}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\delta_\perp}{4}G_{\mu\nu}G^{\mu\nu} + \frac{k}{8}\epsilon^{\mu\nu\lambda}A_\mu G_{\nu\lambda} + \frac{k}{8}\epsilon^{\mu\nu\lambda}C_\mu F_{\nu\lambda}$$

Electric Fields: $E^i = F^{0i}$

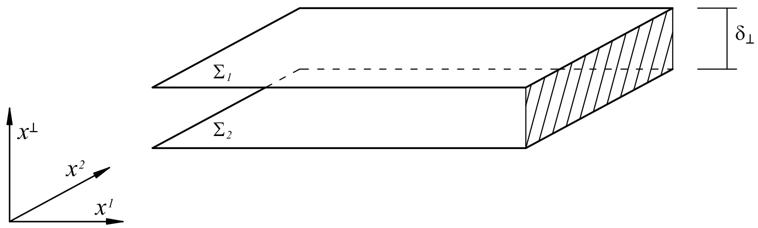
2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



$$\mathcal{L}_3 = -\frac{\delta_\perp}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\delta_\perp}{4} G_{\mu\nu} G^{\mu\nu} + \frac{k}{8} \epsilon^{\mu\nu\lambda} A_\mu G_{\nu\lambda} + \frac{k}{8} \epsilon^{\mu\nu\lambda} C_\mu F_{\nu\lambda}$$

Electric Fields: $\quad E^i = F^{0i} \;\; , \;\; E^\perp = -G_{12}$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction

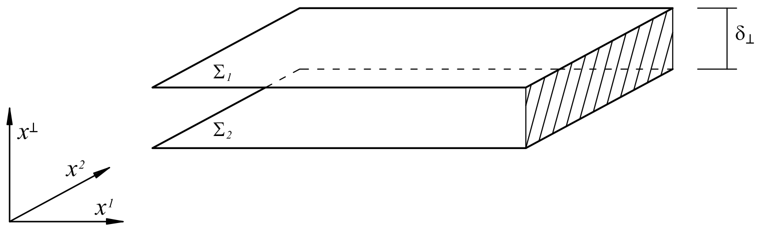


$$\mathcal{L}_3 = -\frac{\delta_\perp}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\delta_\perp}{4} G_{\mu\nu} G^{\mu\nu} + \frac{k}{8} \epsilon^{\mu\nu\lambda} A_\mu G_{\nu\lambda} + \frac{k}{8} \epsilon^{\mu\nu\lambda} C_\mu F_{\nu\lambda}$$

Electric Fields: $E^i = F^{0i} \ , \ \ E^\perp = -G_{12}$

Magnetic Fields:

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction

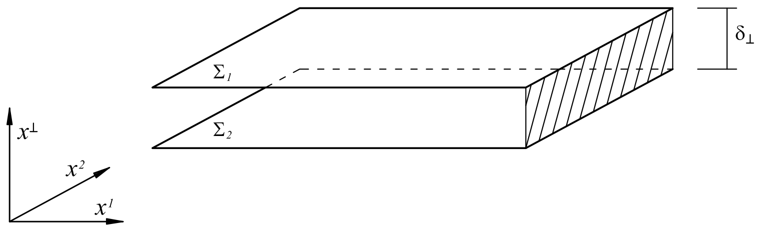


$$\mathcal{L}_3 = -\frac{\delta_\perp}{4} F_{\mu\nu}F^{\mu\nu} + \frac{\delta_\perp}{4} G_{\mu\nu}G^{\mu\nu} + \frac{k}{8}\epsilon^{\mu\nu\lambda}A_\mu G_{\nu\lambda} + \frac{k}{8}\epsilon^{\mu\nu\lambda}C_\mu F_{\nu\lambda}$$

Electric Fields: $E^i = F^{0i} \ , \ E^\perp = -G_{12}$

Magnetic Fields: $B^\perp = F_{12}$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction

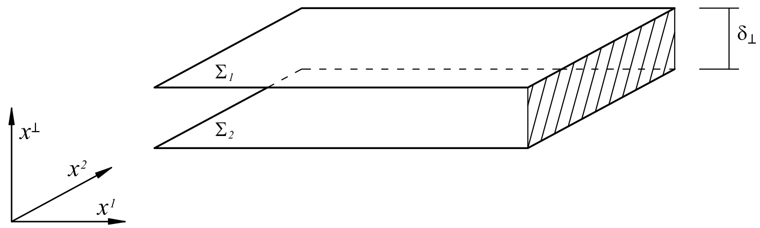


$$\mathcal{L}_3 = -\frac{\delta_\perp}{4} F_{\mu\nu}F^{\mu\nu} + \frac{\delta_\perp}{4} G_{\mu\nu}G^{\mu\nu} + \frac{k}{8}\epsilon^{\mu\nu\lambda}A_\mu G_{\nu\lambda} + \frac{k}{8}\epsilon^{\mu\nu\lambda}C_\mu F_{\nu\lambda}$$

Electric Fields: $E^i = F^{0i} \ , \ E^\perp = -G_{12}$

Magnetic Fields: $B^\perp = F_{12} \ , \ B^i = -G^{0i}$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



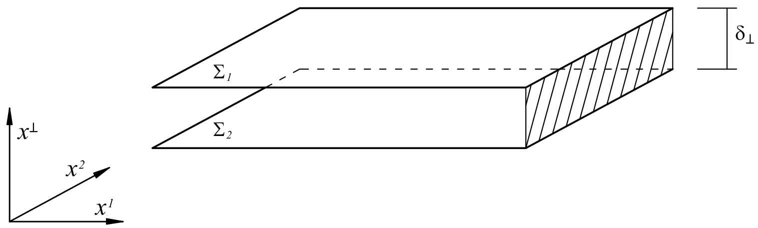
$$\mathcal{L}_3 = -\frac{\delta_\perp}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\delta_\perp}{4} G_{\mu\nu} G^{\mu\nu} + \frac{k}{8} \epsilon^{\mu\nu\lambda} A_\mu G_{\nu\lambda} + \frac{k}{8} \epsilon^{\mu\nu\lambda} C_\mu F_{\nu\lambda}$$

Electric Fields: $E^i = F^{0i} \;,\; E^\perp = -G_{12}$

Magnetic Fields: $B^\perp = F_{12} \;,\; B^i = -G^{0i}$

Canonical Momenta:

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



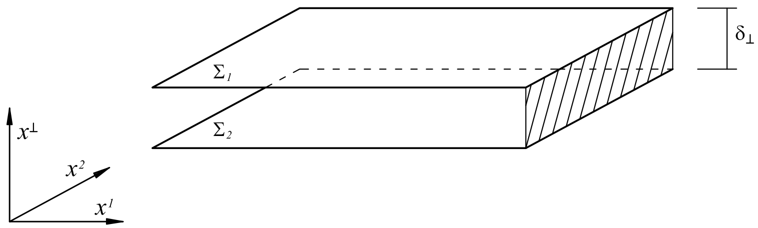
$$\mathcal{L}_3 = -\frac{\delta_\perp}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\delta_\perp}{4} G_{\mu\nu} G^{\mu\nu} + \frac{k}{8} \epsilon^{\mu\nu\lambda} A_\mu G_{\nu\lambda} + \frac{k}{8} \epsilon^{\mu\nu\lambda} C_\mu F_{\nu\lambda}$$

Electric Fields: $E^i = F^{0i} \quad , \quad E^\perp = -G_{12}$

Magnetic Fields: $B^\perp = F_{12} \quad , \quad B^i = -G^{0i}$

Canonical Momenta: $\pi^i_A = -F^{0i} + \dfrac{k}{4\delta_\perp} \epsilon^{ij} C_j$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism – Dimensional Reduction



$$\mathcal{L}_3 = -\frac{\delta_\perp}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\delta_\perp}{4} G_{\mu\nu} G^{\mu\nu} + \frac{k}{8} \epsilon^{\mu\nu\lambda} A_\mu G_{\nu\lambda} + \frac{k}{8} \epsilon^{\mu\nu\lambda} C_\mu F_{\nu\lambda}$$

Electric Fields: $E^i = F^{0i} \quad , \quad E^\perp = -G_{12}$

Magnetic Fields: $B^\perp = F_{12} \quad , \quad B^i = -G^{0i}$

Canonical Momenta:

$$\pi_A^i = -F^{0i} + \frac{k}{4\delta_\perp} \epsilon^{ij} C_j$$

$$\pi_C^i = +G^{0i} + \frac{k}{4\delta_\perp} \epsilon^{ij} A_j$$

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism

- Classically no new results, at the level of the Maxwell equations exactly the same results are obtained.

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism

- Classically no new results, at the level of the Maxwell equations exactly the same results are obtained.

  This is due to the physical electromagnetic field definitions.

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism

- Classically no new results, at the level of the Maxwell equations exactly the same results are obtained.

  This is due to the physical electromagnetic field definitions.

- However at variational level we have a new theory which solves the inconsistency concerning the Bianchi Identities.

2. Extended $U_e(1) \times U_g(1)$ Electromagnetism

- Classically no new results, at the level of the Maxwell equations exactly the same results are obtained.

  This is due to the physical electromagnetic field definitions.

- However at variational level we have a new theory which solves the inconsistency concerning the Bianchi Identities.

- In planar systems we will obtain new observable consequences when considering Extended $U_e(1) \times U_g(1)$ Electromagnetism.

3. Functional Quantization in Planar Systems

3. Functional Quantization in Planar Systems

Main motivation is application to the fractional Hall effect

3. Functional Quantization in Planar Systems

Main motivation is application to the fractional Hall effect , in particular to obtain a *microscopical* description.

3. Functional Quantization in Planar Systems

Main motivation is application to the fractional Hall effect , in particular to obtain a *microscopical* description.

From an effective Landau-Ginzburg theory:

3. Functional Quantization in Planar Systems

Main motivation is application to the fractional Hall effect , in particular to obtain a *microscopical* description.

From an effective Landau-Ginzburg theory: $\mathcal{L} = \mathcal{L}_3 + \mathcal{L}_{\phi,C}$

3. Functional Quantization in Planar Systems

Main motivation is application to the fractional Hall effect , in particular to obtain a *microscopical* description.

From an effective Landau-Ginzburgh theory: $\mathcal{L} = \mathcal{L}_3 + \mathcal{L}_{\phi,C}$

$$
\begin{aligned}
\mathcal{L}_{\phi,C} &= -\phi^* \left[i\partial_0 - e\alpha A_0 - g\hat{\beta}C_0 \right] \phi \\
&\quad -\frac{1}{2\bar{m}}\phi^* \left[-i\nabla - e\alpha \mathbf{A} - g\hat{\beta}\mathbf{C} \right]^2 \phi \\
&\quad +\mu\phi^*\phi - \lambda(\phi^*\phi)^2 \ .
\end{aligned}
$$

3. Functional Quantization in Planar Systems

Main motivation is application to the fractional Hall effect , in particular to obtain a *microscopical* description.

From an effective Landau-Ginzburgh theory: $\mathcal{L} = \mathcal{L}_3 + \mathcal{L}_{\phi,C}$

$$\begin{aligned}
\mathcal{L}_{\phi,C} &= -\phi^* \left[i\partial_0 - e\alpha A_0 - g\hat{\beta}C_0 \right] \phi \\
&\quad -\frac{1}{2\bar{m}}\phi^* \left[-i\nabla - e\alpha \mathbf{A} - g\hat{\beta}\mathbf{C} \right]^2 \phi \\
&\quad +\mu\phi^*\phi - \lambda(\phi^*\phi)^2 \ .
\end{aligned}$$

α: electric vortex density , $\hat{\beta}$: magnetic vortex density

3. Functional Quantization in Planar Systems

Main motivation is application to the fractional Hall effect , in particular to obtain a *microscopical* description.

From an effective Landau-Ginzburgh theory: $\mathcal{L} = \mathcal{L}_3 + \mathcal{L}_{\phi,C}$

it is obtained a orthogonal electric field:

3. Functional Quantization in Planar Systems

Main motivation is application to the fractional Hall effect , in particular to obtain a *microscopical* description.

From an effective Landau-Ginzburgh theory: $\mathcal{L} = \mathcal{L}_3 + \mathcal{L}_{\phi,C}$

it is obtained a orthogonal electric field:

$$\tilde{E} = -\frac{e\alpha}{g\hat{\beta}}B$$

3. Functional Quantization in Planar Systems

Main motivation is application to the fractional Hall effect , in particular to obtain a *microscopical* description.

From an effective Landau-Ginzburgh theory: $\mathcal{L} = \mathcal{L}_3 + \mathcal{L}_{\phi,C}$

it is obtained a orthogonal electric field:

$$\tilde{E} = -\frac{e\alpha}{g\hat{\beta}}B$$

and the Hall conductance is:

3. Functional Quantization in Planar Systems

Main motivation is application to the fractional Hall effect , in particular to obtain a *microscopical* description.

From an effective Landau-Ginzburgh theory: $\mathcal{L} = \mathcal{L}_3 + \mathcal{L}_{\phi,C}$

it is obtained a orthogonal electric field:

$$\tilde{E} = -\frac{e\alpha}{g\hat{\beta}}B$$

and the Hall conductance is:

$$j^i = \frac{e\alpha}{2g\hat{\beta}}\epsilon^{ij}E_j$$

3. Functional Quantization in Planar Systems

Main motivation is application to the fractional Hall effect , in particular to obtain a *microscopical* description.

From an effective Landau-Ginzburgh theory: $\mathcal{L} = \mathcal{L}_3 + \mathcal{L}_{\phi,C}$

it is obtained a orthogonal electric field:

$$\tilde{E} = -\frac{e\alpha}{g\hat{\beta}}B$$

and the Hall conductance is:

$$j^i = \frac{e\alpha}{2g\hat{\beta}}\epsilon^{0ij}E_j$$

3. Functional Quantization in Planar Systems

Main motivation is application to the fractional Hall effect , in particular to obtain a *microscopical* description.

From an effective Landau-Ginzburgh theory: $\mathcal{L} = \mathcal{L}_3 + \mathcal{L}_{\phi,C}$

it is obtained a orthogonal electric field:

$$\tilde{E} = -\frac{e\alpha}{g\hat{\beta}}B$$

and the Hall conductance is:

$$j^i = \frac{e\alpha}{2g\hat{\beta}}\epsilon^{0ij}E_j = \hat{\sigma}_H\epsilon^{0ij}E_j$$

3. Functional Quantization in Planar Systems

Main motivation is application to the fractional Hall effect , in particular to obtain a *microscopical* description.

From an effective Landau-Ginzburgh theory: $\mathcal{L} = \mathcal{L}_3 + \mathcal{L}_{\phi,C}$

it is obtained a orthogonal electric field:

$$\tilde{E} = -\frac{e\alpha}{g\hat{\beta}}B$$

and the Hall conductance is:

$$j^i = \frac{e\alpha}{2g\hat{\beta}}\epsilon^{0ij}E_j = \hat{\sigma}_H\epsilon^{0ij}E_j$$

Φ_0 quantization due to Dirac's condition :

3. Functional Quantization in Planar Systems

Main motivation is application to the fractional Hall effect , in particular to obtain a *microscopical* description.

From an effective Landau-Ginzburgh theory: $\mathcal{L} = \mathcal{L}_3 + \mathcal{L}_{\phi,C}$

it is obtained a orthogonal electric field:

$$\tilde{E} = -\frac{e\alpha}{g\hat{\beta}} B$$

and the Hall conductance is:

$$j^i = \frac{e\alpha}{2g\hat{\beta}}\epsilon^{0ij}E_j = \hat{\sigma}_H \epsilon^{0ij} E_j$$

Φ_0 quantization due to Dirac's condition :

$$\hat{\sigma}_H = \frac{e\alpha}{2g\hat{\beta}} = \frac{e}{\Phi_0}\frac{\alpha}{\hat{\beta}}$$

3. Functional Quantization in Planar Systems

Main novelties of the model:

3. Functional Quantization in Planar Systems

Main novelties of the model:

- macroscopical P and T invariance in the planar system.

3. Functional Quantization in Planar Systems

Main novelties of the model:

- macroscopical P and T invariance in the planar system.
- solves inconsistence in the vector/pseudo-vector nature of the electromagnetic equations.

3. Functional Quantization in Planar Systems

Main novelties of the model:

- macroscopical P and T invariance in the planar system.
- solves inconsistence in the vector/pseudo-vector nature of the electromagnetic equations.
- theoretical justification for the experimentally measured fractional charge $e^* = \frac{1}{2n+1}$ for every filling fraction $\nu = \frac{p}{2n+1}$ independently of p.

3. Functional Quantization in Planar Systems

Main novelties of the model:

- macroscopical P and T invariance in the planar system.
- solves inconsistence in the vector/pseudo-vector nature of the electromagnetic equations.
- theoretical justification for the experimentally measured fractional charge $e^* = \frac{1}{2n+1}$ for every filling fraction $\nu = \frac{p}{2n+1}$ independently of p.
- theoretical justification for the low energy contribution to Laughlin's wave function solutions due to the negative energy contributions of pseudo-photon excitations (which are ghost or phantoms).

Main novelties of the model:

3. Functional Quantization in Planar Systems

Main novelties of the model:

- equivalence between Dirac's quantization condition and the experimentally verified quantization of magnetic flux given directly in terms of the units charges e and g which is fully justified in the context of $U_e(1) \times U_g(1)$ electromagnetism.

3. Functional Quantization in Planar Systems

Main novelties of the model:

- equivalence between Dirac's quantization condition and the experimentally verified quantization of magnetic flux given directly in terms of the units charges e and g which is fully justified in the context of $U_e(1) \times U_g(1)$ electromagnetism.
- theoretical justification for the orthogonal electric potential due to pseudo-photon electric vortexes which may justify the experimental existence of BEC condensates in bi-layer electron-electron Hall systems instead of its existence in electron-hole Hall systems as originally expected.

3. Functional Quantization in Planar Systems

3. Functional Quantization in Planar Systems

$$\mathcal{L}_3 = -\frac{\delta_\perp}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\delta_\perp}{4} G_{\mu\nu} G^{\mu\nu} + \frac{k}{8} \epsilon^{\mu\nu\lambda} A_\mu G_{\nu\lambda} + \frac{k}{8} \epsilon^{\mu\nu\lambda} C_\mu F_{\nu\lambda}$$

3. Functional Quantization in Planar Systems

$$\mathcal{L}_3 = -\frac{\delta_\perp}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\delta_\perp}{4} G_{\mu\nu} G^{\mu\nu} + \frac{k}{8} \epsilon^{\mu\nu\lambda} A_\mu G_{\nu\lambda} + \frac{k}{8} \epsilon^{\mu\nu\lambda} C_\mu F_{\nu\lambda}$$

$$\pi_A^i = -F^{0i} + \frac{k}{4\delta_\perp} \epsilon^{ij} C_j$$
$$\pi_C^i = +G^{0i} + \frac{k}{4\delta_\perp} \epsilon^{ij} A_j$$

3. Functional Quantization in Planar Systems

$$\pi^i_A = -F^{0i} + \frac{k}{4\delta_\perp}\epsilon^{ij}C_j \qquad \pi^i_C = +G^{0i} + \frac{k}{4\delta_\perp}\epsilon^{ij}A_j$$

3. Functional Quantization in Planar Systems

$$\pi^i_A = -F^{0i} + \frac{k}{4\delta_\perp}\epsilon^{ij}C_j \qquad \pi^i_C = +G^{0i} + \frac{k}{4\delta_\perp}\epsilon^{ij}A_j$$

Defining Hamiltonian as usual: $\mathcal{H}_{AC} = +\pi^i_A \partial_0 A_i + \pi^i_C \partial_0 C_i - \mathcal{L}_3$

3. Functional Quantization in Planar Systems

$$\pi_A^i = -F^{0i} + \frac{k}{4\delta_\perp}\epsilon^{ij}C_j \qquad \pi_C^i = +G^{0i} + \frac{k}{4\delta_\perp}\epsilon^{ij}A_j$$

Defining Hamiltonian as usual: $\mathcal{H}_{AC} = +\pi_A^i\partial_0 A_i + \pi_C^i\partial_0 C_i - \mathcal{L}_3$

and considering functional Schrödinger representation:

3. Functional Quantization in Planar Systems

$$\pi^i_A = -F^{0i} + \frac{k}{4\delta_\perp}\epsilon^{ij}C_j \qquad \pi^i_C = +G^{0i} + \frac{k}{4\delta_\perp}\epsilon^{ij}A_j$$

Defining Hamiltonian as usual: $\mathcal{H}_{AC} = +\pi^i_A\partial_0 A_i + \pi^i_C\partial_0 C_i - \mathcal{L}_3$

and considering functional Schrödinger representation:

$$\pi^i_A = -i\,\frac{\delta}{\delta A_i} \qquad \pi^i_C = +i\,\frac{\delta}{\delta C_i}$$

3. Functional Quantization in Planar Systems

$$\pi^i_A = -F^{0i} + \frac{k}{4\delta_\perp}\epsilon^{ij}C_j \qquad \pi^i_C = +G^{0i} + \frac{k}{4\delta_\perp}\epsilon^{ij}A_j$$

Defining Hamiltonian as usual: $\mathcal{H}_{AC} = +\pi^i_A\partial_0 A_i + \pi^i_C\partial_0 C_i - \mathcal{L}_3$

and considering functional Schrödinger representation:

$$\pi^i_A = -i\,\frac{\delta}{\delta A_i} \qquad \pi^i_C = +i\,\frac{\delta}{\delta C_i}$$

we obtain 3 functional constraints:

3. Functional Quantization in Planar Systems

$$\pi^i_A = -F^{0i} + \frac{k}{4\delta_\perp}\epsilon^{ij}C_j \qquad \pi^i_C = +G^{0i} + \frac{k}{4\delta_\perp}\epsilon^{ij}A_j$$

Defining Hamiltonian as usual: $\mathcal{H}_{AC} = +\pi^i_A\partial_0 A_i + \pi^i_C\partial_0 C_i - \mathcal{L}_3$

and considering functional Schrödinger representation:

$$\pi^i_A = -i\,\frac{\delta}{\delta A_i} \qquad \pi^i_C = +i\,\frac{\delta}{\delta C_i}$$

we obtain 3 functional constraints:

$$\hat{\mathcal{G}}_A\Phi_{(0,0)}[A,C] = \left[\partial_i\left(i\frac{\delta}{\delta A_i} - \frac{k}{4\delta_\perp}\epsilon^{ij}C_j\right)\right]\Phi_{(0,0)}[A,C] = 0$$

3. Functional Quantization in Planar Systems

$$\pi_A^i = -F^{0i} + \frac{k}{4\delta_\perp}\epsilon^{ij}C_j \qquad \pi_C^i = +G^{0i} + \frac{k}{4\delta_\perp}\epsilon^{ij}A_j$$

Defining Hamiltonian as usual: $\mathcal{H}_{AC} = +\pi_A^i\partial_0 A_i + \pi_C^i\partial_0 C_i - \mathcal{L}_3$

and considering functional Schrödinger representation:

$$\pi_A^i = -i\,\frac{\delta}{\delta A_i} \qquad\qquad \pi_C^i = +i\,\frac{\delta}{\delta C_i}$$

we obtain 3 functional constraints:

$$\hat{\mathcal{G}}_A\Phi_{(0,0)}[A,C] = \left[\partial_i\left(i\frac{\delta}{\delta A_i} - \frac{k}{4\delta_\perp}\epsilon^{ij}C_j\right)\right]\Phi_{(0,0)}[A,C] = 0$$

$$\hat{\mathcal{G}}_C\Phi_{(0,0)}[A,C] = \left[\partial_i\left(i\frac{\delta}{\delta C_i} + \frac{k}{4\delta_\perp}\epsilon^{ij}A_j\right)\right]\Phi_{(0,0)}[A,C] = 0$$

3. Functional Quantization in Planar Systems

we obtain 3 functional constraints:

$$\hat{\mathcal{G}}_A \Phi_{(0,0)}[A, C] = \left[\partial_i \left(i \frac{\delta}{\delta A_i} - \frac{k}{4\delta_\perp} \epsilon^{ij} C_j \right) \right] \Phi_{(0,0)}[A, C] = 0$$

$$\hat{\mathcal{G}}_C \Phi_{(0,0)}[A, C] = \left[\partial_i \left(i \frac{\delta}{\delta C_i} + \frac{k}{4\delta_\perp} \epsilon^{ij} A_j \right) \right] \Phi_{(0,0)}[A, C] = 0$$

3. Functional Quantization in Planar Systems

we obtain 3 functional constraints:

$$\hat{\mathcal{G}}_A \Phi_{(0,0)}[A,C] = \left[\partial_i \left(i\frac{\delta}{\delta A_i} - \frac{k}{4\delta_\perp}\epsilon^{ij} C_j \right)\right] \Phi_{(0,0)}[A,C] = 0$$

$$\hat{\mathcal{G}}_C \Phi_{(0,0)}[A,C] = \left[\partial_i \left(i\frac{\delta}{\delta C_i} + \frac{k}{4\delta_\perp}\epsilon^{ij} A_j \right)\right] \Phi_{(0,0)}[A,C] = 0$$

$$\begin{aligned} \hat{\mathcal{H}}_{AC} \Phi_{(0,0)}[A,C] &= \left[+\frac{1}{2}\left(i\frac{\delta}{\delta A_i} + \frac{k}{4\delta_\perp}\epsilon^{ij} C_j \right)\left(i\frac{\delta}{\delta A_i} + \frac{k}{4\delta_\perp}\epsilon^{ik} C_k \right) \right. \\ &\quad -\frac{1}{2}\left(i\frac{\delta}{\delta C_i} - \frac{k}{4\delta_\perp}\epsilon^{ij} A_j \right)\left(i\frac{\delta}{\delta C_i} - \frac{k}{4\delta_\perp}\epsilon^{ik} A_k \right) \\ &\quad \left. +\frac{1}{4} F^{ij} F^{ij} - \frac{1}{4} G^{ij} G^{ij} \right] \Phi_{(0,0)}[A,C] \\ &= \mathcal{E}_{(0,0)} \Phi_{(0,0)}[A,C] \end{aligned}$$

3. Functional Quantization in Planar Systems

we obtain 3 functional constraints:

$$\hat{\mathcal{G}}_A \Phi_{(0,0)}[A,C] = \left[\partial_i \left(i\frac{\delta}{\delta A_i} - \frac{k}{4\delta_\perp}\epsilon^{ij} C_j \right)\right] \Phi_{(0,0)}[A,C] = 0$$

$$\hat{\mathcal{G}}_C \Phi_{(0,0)}[A,C] = \left[\partial_i \left(i\frac{\delta}{\delta C_i} + \frac{k}{4\delta_\perp}\epsilon^{ij} A_j \right)\right] \Phi_{(0,0)}[A,C] = 0$$

Wave functional solution for both Gauss' laws is:

$$\Phi_{(0,0)}[A,C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i C_j}$$

3. Functional Quantization in Planar Systems

we obtain 3 functional constraints:

$$\hat{\mathcal{G}}_A \Phi_{(0,0)}[A,C] = \left[\partial_i \left(i\frac{\delta}{\delta A_i} - \frac{k}{4\delta_\perp}\epsilon^{ij} C_j \right)\right] \Phi_{(0,0)}[A,C] = 0$$

$$\hat{\mathcal{G}}_C \Phi_{(0,0)}[A,C] = \left[\partial_i \left(i\frac{\delta}{\delta C_i} + \frac{k}{4\delta_\perp}\epsilon^{ij} A_j \right)\right] \Phi_{(0,0)}[A,C] = 0$$

Wave functional solution for both Gauss' laws is:

$$\Phi_{(0,0)}[A,C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i C_j}$$

which corresponds to the topological ground-state

3. Functional Quantization in Planar Systems

$$\Phi_{(0,0)}[A, C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i C_j}$$

3. Functional Quantization in Planar Systems

$$\Phi_{(0,0)}[A, C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i C_j}$$

Hamiltonian constraint reads:

3. Functional Quantization in Planar Systems

$$\Phi_{(0,0)}[A, C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i C_j}$$

Hamiltonian constraint reads:

$$\frac{1}{4}\int dx^2 \left(F^{ij}F^{ij} - \frac{k^2}{2\delta_\perp^2}A^i A^i - G^{ij}G^{ij} + \frac{k^2}{2\delta_\perp^2}C^i C^i\right)\Phi_{(0,0)}[A, C]$$
$$= \mathcal{E}_{(0,0)}\Phi_{(0,0)}[A, C]$$

3. Functional Quantization in Planar Systems

$$\Phi_{(0,0)}[A,C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i C_j}$$

Hamiltonian constraint reads:

$$\frac{1}{4}\int dx^2 \left(F^{ij}F^{ij} - \frac{k^2}{2\delta_\perp^2} A^i A^i - G^{ij}G^{ij} + \frac{k^2}{2\delta_\perp^2} C^i C^i \right) \Phi_{(0,0)}[A,C]$$
$$= \mathcal{E}_{(0,0)} \Phi_{(0,0)}[A,C]$$

Energy minimized by specific classical configurations:

3. Functional Quantization in Planar Systems

$$\Phi_{(0,0)}[A,C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i C_j}$$

Hamiltonian constraint reads:

$$\frac{1}{4}\int dx^2 \left(F^{ij}F^{ij} - \frac{k^2}{2\delta_\perp^2} A^i A^i - G^{ij}G^{ij} + \frac{k^2}{2\delta_\perp^2} C^i C^i \right) \Phi_{(0,0)}[A,C]$$
$$= \mathcal{E}_{(0,0)} \Phi_{(0,0)}[A,C]$$

Energy minimized by specific classical configurations:

either the trivial solutions $A = C = 0$ or

3. Functional Quantization in Planar Systems

$$\Phi_{(0,0)}[A,C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i C_j}$$

Hamiltonian constraint reads:

$$\frac{1}{4}\int dx^2 \left(F^{ij}F^{ij} - \frac{k^2}{2\delta_\perp^2} A^i A^i - G^{ij}G^{ij} + \frac{k^2}{2\delta_\perp^2} C^i C^i \right) \Phi_{(0,0)}[A,C]$$
$$= \mathcal{E}_{(0,0)} \Phi_{(0,0)}[A,C]$$

Energy minimized by specific classical configurations:

the solutions of the functional equations

3. Functional Quantization in Planar Systems

$$\Phi_{(0,0)}[A,C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i C_j}$$

Hamiltonian constraint reads:

$$\frac{1}{4}\int dx^2 \left(F^{ij}F^{ij} - \frac{k^2}{2\delta_\perp^2} A^i A^i - G^{ij}G^{ij} + \frac{k^2}{2\delta_\perp^2} C^i C^i \right) \Phi_{(0,0)}[A,C]$$
$$= \mathcal{E}_{(0,0)} \Phi_{(0,0)}[A,C]$$

Energy minimized by specific classical configurations:

the solutions of the functional equations

$$\int dx^2 \, F_{ij}F^{ij} = \frac{k^2}{2\delta_\perp^2} \int dx^2 \, A_i A^i \, , \quad \int dx^2 \, G_{ij}G^{ij} = \frac{k^2}{2\delta_\perp^2} \int dx^2 \, C_i C^i$$

3. Functional Quantization in Planar Systems

$$\Phi_{(0,0)}[A, C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i C_j}$$

Hamiltonian constraint reads:

$$\frac{1}{4}\int dx^2 \left(F^{ij}F^{ij} - \frac{k^2}{2\delta_\perp^2} A^i A^i - G^{ij}G^{ij} + \frac{k^2}{2\delta_\perp^2} C^i C^i\right) \Phi_{(0,0)}[A, C]$$
$$= \mathcal{E}_{(0,0)} \Phi_{(0,0)}[A, C]$$

Energy minimized by specific classical configurations:

i.e. the vortex solutions

3. Functional Quantization in Planar Systems

$$\Phi_{(0,0)}[A,C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i C_j}$$

Hamiltonian constraint reads:

$$\frac{1}{4}\int dx^2 \left(F^{ij}F^{ij} - \frac{k^2}{2\delta_\perp^2} A^i A^i - G^{ij}G^{ij} + \frac{k^2}{2\delta_\perp^2} C^i C^i\right) \Phi_{(0,0)}[A,C]$$
$$= \mathcal{E}_{(0,0)} \Phi_{(0,0)}[A,C]$$

Energy minimized by specific classical configurations:

i.e. the vortex solutions $\left(\text{holding } \mathcal{E}_{(0,0)} = 0\right)$

3. Functional Quantization in Planar Systems

$$\Phi_{(0,0)}[A,C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i C_j}$$

Hamiltonian constraint reads:

$$\frac{1}{4}\int dx^2 \left(F^{ij}F^{ij} - \frac{k^2}{2\delta_\perp^2} A^i A^i - G^{ij}G^{ij} + \frac{k^2}{2\delta_\perp^2} C^i C^i \right) \Phi_{(0,0)}[A,C]$$
$$= \mathcal{E}_{(0,0)} \Phi_{(0,0)}[A,C]$$

Energy minimized by specific classical configurations:

i.e. the vortex solutions $\left(\text{holding } \mathcal{E}_{(0,0)} = 0\right)$

$$A^i = \pm \frac{\epsilon^{ij}}{k^2} \frac{r_j - \bar{r}_j}{|r - \bar{r}|} \qquad C^i = \pm \frac{\epsilon^{ij}}{k^2} \frac{r_j - \bar{r}_j}{|r - \bar{r}|}$$

3. Functional Quantization in Planar Systems

$$A^i = \pm\epsilon^{ij}\frac{r_j - \bar{r}_j}{|r - \bar{r}|} \qquad C^i = \pm\epsilon^{ij}\frac{r_j - \bar{r}_j}{|r - \bar{r}|}$$

3. Functional Quantization in Planar Systems

$$A^i = \pm\epsilon^{ij}\frac{r_j - \bar{r}_j}{|r - \bar{r}|} \qquad C^i = \pm\epsilon^{ij}\frac{r_j - \bar{r}_j}{|r - \bar{r}|}$$

These solutions require a vortex radius of:

3. Functional Quantization in Planar Systems

$$A^i = \pm\epsilon^{ij}\frac{r_j - \bar{r}_j}{|r - \bar{r}|} \qquad C^i = \pm\epsilon^{ij}\frac{r_j - \bar{r}_j}{|r - \bar{r}|}$$

These solutions require a vortex radius of:

$$R = \sqrt{2\pi}\delta_\perp$$

3. Functional Quantization in Planar Systems

$$A^i = \pm\epsilon^{ij}\frac{r_j - \bar{r}_j}{|r - \bar{r}|} \qquad C^i = \pm\epsilon^{ij}\frac{r_j - \bar{r}_j}{|r - \bar{r}|}$$

These solutions require a vortex radius of:

$$R = \sqrt{2\pi}\delta_\perp$$

which physically relates the system thickness to the planar magnetic length:

3. Functional Quantization in Planar Systems

$$A^i = \pm\epsilon^{ij}\frac{r_j - \bar{r}_j}{|r - \bar{r}|} \qquad C^i = \pm\epsilon^{ij}\frac{r_j - \bar{r}_j}{|r - \bar{r}|}$$

These solutions require a vortex radius of:

$$R = \sqrt{2\pi}\delta_\perp$$

which physically relates the system thickness to the planar magnetic length:

$$\delta_\perp = \frac{l_m}{\sqrt{\pi}}$$

3. Functional Quantization in Planar Systems

3. Functional Quantization in Planar Systems

By considering the lowering and raising operators:

3. Functional Quantization in Planar Systems

By considering the lowering and raising operators:

$$\hat{E}_z = -i\frac{\delta}{\delta A_z} + \frac{ik}{4}C_{\bar{z}} \quad , \quad \hat{E}_{\bar{z}} = -i\frac{\delta}{\delta A_{\bar{z}}} - \frac{ik}{4}C_z$$

By considering the lowering and raising operators:

$$\hat{E}_z = -i\frac{\delta}{\delta A_z} + \frac{ik}{4}C_{\bar{z}} \quad , \quad \hat{E}_{\bar{z}} = -i\frac{\delta}{\delta A_{\bar{z}}} - \frac{ik}{4}C_z$$

$$\hat{B}_z = +i\frac{\delta}{\delta C_z} + \frac{ik}{4}A_{\bar{z}} \quad , \quad \hat{B}_{\bar{z}} = +i\frac{\delta}{\delta C_{\bar{z}}} - \frac{ik}{4}A_z$$

3. Functional Quantization in Planar Systems

By considering the lowering and raising operators:

$$\hat{E}_z = -i\frac{\delta}{\delta A_z} + \frac{ik}{4}C_{\bar{z}} \quad , \quad \hat{E}_{\bar{z}} = -i\frac{\delta}{\delta A_{\bar{z}}} - \frac{ik}{4}C_z$$

$$\hat{B}_z = +i\frac{\delta}{\delta C_z} + \frac{ik}{4}A_{\bar{z}} \quad , \quad \hat{B}_{\bar{z}} = +i\frac{\delta}{\delta C_{\bar{z}}} - \frac{ik}{4}A_z$$

We can build the excited wave functionals:

3. Functional Quantization in Planar Systems

By considering the lowering and raising operators:

$$\hat{E}_z = -i\frac{\delta}{\delta A_z} + \frac{ik}{4}C_{\bar{z}} \quad , \quad \hat{E}_{\bar{z}} = -i\frac{\delta}{\delta A_{\bar{z}}} - \frac{ik}{4}C_z$$

$$\hat{B}_z = +i\frac{\delta}{\delta C_z} + \frac{ik}{4}A_{\bar{z}} \quad , \quad \hat{B}_{\bar{z}} = +i\frac{\delta}{\delta C_{\bar{z}}} - \frac{ik}{4}A_z$$

We can build the excited wave functionals:

$$\Phi_{(n,m)} = \left(\hat{E}_{\bar{z}}\right)^n \left(\hat{B}_{\bar{z}}\right)^m \Phi_{(0,0)}[A, C]$$

3. Functional Quantization in Planar Systems

By considering the lowering and raising operators:

$$\hat{E}_z = -i\frac{\delta}{\delta A_z} + \frac{ik}{4}C_{\bar{z}} \quad , \quad \hat{E}_{\bar{z}} = -i\frac{\delta}{\delta A_{\bar{z}}} - \frac{ik}{4}C_z$$

$$\hat{B}_z = +i\frac{\delta}{\delta C_z} + \frac{ik}{4}A_{\bar{z}} \quad , \quad \hat{B}_{\bar{z}} = +i\frac{\delta}{\delta C_{\bar{z}}} - \frac{ik}{4}A_z$$

We can build the excited wave functionals:

$$\Phi_{(n,m)} = \left(\hat{E}_{\bar{z}}\right)^n \left(\hat{B}_{\bar{z}}\right)^m \Phi_{(0,0)}[A, C]$$

Seems consistent with Laughlin's wave functions

3. Functional Quantization in Planar Systems

By considering the lowering and raising operators:

$$\hat{E}_z = -i\frac{\delta}{\delta A_z} + \frac{ik}{4}C_{\bar{z}} \quad , \quad \hat{E}_{\bar{z}} = -i\frac{\delta}{\delta A_{\bar{z}}} - \frac{ik}{4}C_z$$

$$\hat{B}_z = +i\frac{\delta}{\delta C_z} + \frac{ik}{4}A_{\bar{z}} \quad , \quad \hat{B}_{\bar{z}} = +i\frac{\delta}{\delta C_{\bar{z}}} - \frac{ik}{4}A_z$$

We can build the excited wave functionals:

$$\Phi_{(n,m)} = \left(\hat{E}_{\bar{z}}\right)^n \left(\hat{B}_{\bar{z}}\right)^m \Phi_{(0,0)}[A, C]$$

Seems consistent with Laughlin's wave functions although it is still missing a direct proof (in progress).

3. Functional Quantization in Planar Systems

3. Functional Quantization in Planar Systems

Considering external fields:

3. Functional Quantization in Planar Systems

Considering external fields: $A_\mu = A_\mu^{\rm ext} + a_\mu$

3. Functional Quantization in Planar Systems

Considering external fields: $A_\mu = A_\mu^{\rm ext} + a_\mu$

Wave functional solution

3. Functional Quantization in Planar Systems

Considering external fields: $A_\mu = A_\mu^{\rm ext} + a_\mu$

Wave functional solution:

$$\Psi_A[a, C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij}\left(A_i^{\rm ext}+a_i\right)C_j}$$

3. Functional Quantization in Planar Systems

Considering external fields: $A_\mu = A_\mu^{\rm ext} + a_\mu$

Wave functional solution is a correction to $\Phi_{(0,0)}$:

$$\Psi_A[a, C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i^{\rm ext} C_j}\,\Phi_{(0,0)}$$

3. Functional Quantization in Planar Systems

Considering external fields: $A_\mu = A_\mu^{\mathrm{ext}} + a_\mu$

Wave functional solution is a correction to $\Phi_{(0,0)}$:

$$\Psi_A[a, C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i^{\mathrm{ext}} C_j}\, \Phi_{(0,0)}$$

These may be traced back to a sum over equivalent *excited* wave functionals,

3. Functional Quantization in Planar Systems

Considering external fields: $A_\mu = A_\mu^{\rm ext} + a_\mu$

Wave functional solution is a correction to $\Phi_{(0,0)}$:

$$\Psi_A[a, C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i^{\rm ext} C_j}\,\Phi_{(0,0)}$$

These may be traced back to a sum over equivalent *excited* wave functionals, shown by considering a measure-shift in the path integral $a \to a - A^{\rm ext}$ and integrating the a field:

3. Functional Quantization in Planar Systems

Considering external fields: $A_\mu = A_\mu^{\rm ext} + a_\mu$

Wave functional solution is a correction to $\Phi_{(0,0)}$:

$$\Psi_A[a, C] = e^{-i\frac{k}{4\delta_\perp}\int dx^2 \epsilon^{ij} A_i^{\rm ext} C_j}\,\Phi_{(0,0)}$$

These may be traced back to a sum over equivalent *excited* wave functionals, shown by considering a measure-shift in the path integral $a \to a - A^{\rm ext}$ and integrating the a field:

$$\Psi_A[C] = \left(1 - i\frac{k}{4\delta_\perp}\epsilon^{ij} A_i^{\rm ext} C_j + \ldots\right)\Phi_0[C]$$

3. Functional Quantization in Planar Systems

3. Functional Quantization in Planar Systems

- We have shown that Pseudo-Photon theories hold new results in planar systems

3. Functional Quantization in Planar Systems

- We have shown that Pseudo-Photon theories hold new results in planar systems
- And that the vortex solutions are justified in a functional framework

3. Functional Quantization in Planar Systems

- We have shown that Pseudo-Photon theories hold new results in planar systems
- And that the vortex solutions are justified in a functional framework
- Still in progress a derivation of fractional statistics for anions

3. Functional Quantization in Planar Systems

- We have shown that Pseudo-Photon theories hold new results in planar systems
- And that the vortex solutions are justified in a functional framework
- Still in progress a derivation of fractional statistics for anions

Other relevant frameworks for Pseudo-Photon theories are plasmas and radiative corrections in strong non-regular electromagnetic fields.

3. Functional Quantization in Planar Systems

- We have shown that Pseudo-Photon theories hold new results in planar systems
- And that the vortex solutions are justified in a functional framework
- Still in progress a derivation of fractional statistics for anions

Other relevant frameworks for Pseudo-Photon theories are plasmas and radiative corrections in strong non-regular electromagnetic fields.

But that is another story...