ELLIPTIC AND PARABOLIC EQUATIONS WITH NONSTANDARD GROWTH CONDITIONS

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Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with Lipschitz-continuous boundary Γ and $Q = \Omega \times (0,T]$. We study the degenerate parabolic equations with anisotropic nonlinearity and nonstandard growth conditions

$$u_t - \sum_i D_i \left[a_i(z,u)|D_i u|^{p_i(z)-2} D_i u + b_i(z,u) \right] + \sum_i d_i(z,u) D_i u + d(z,u) = 0, \qquad (1)$$

$z = (x,t) \in Q = \Omega \times (0,T]$, under the boundary and initial conditions

$$u = 0 \text{ on } \partial\Omega \times [0,T], \qquad u(x,0) = u_0(x) \text{ in } \Omega. \qquad (2)$$

We consider also corresponding elliptic equations (1) with $z = x, u_t = 0$ under the Dirichlet boundary conditions. The coefficients $\alpha_i, p_i, a_i,\ p_i,\ b_i,\ d_i,\ d$ are given functions of their arguments. Such equations occur in the mathematical modeling of various physical processes, e.g., the flows of electro-rheological fluids or fluids with temperature–dependent viscosity, processes of filtration through inhomogeneous anisotropic media. We give a classification of weak solutions of such problems. We prove the existence and uniqueness of weak energy solutions of problems (1), (2) and study the localization (vanishing) properties of such solutions for elliptic and parabolic cases. Using a modification of the method of local energy estimates presented in [1] we show that solutions of elliptic equation (1) may identically vanish on a set of nonzero measure either due to a suitable diffusion–absorption balance, or because of strong anisotropy of the diffusion operator. We also consider the questions of homogenization for such elliptic equations. For parabolic equations we prove the localization (vanishing) properties of energy solutions such that extinction in finite time and finite speed of propagation of disturbances. The presentation follows papers [2; 3; 4; 5].

REFERENCES

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