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WHAT IS THE COMPLEXITY OF WEAKLY SINGULAR INTEGRAL EQUATIONS?

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Consider the integral equation

$$u(x) = \int_0^1 (a(x,y) \mid x-y \mid^{-\nu} + b(x,y))u(y)dy + f(x), \;\; 0 \le x \le 1, \tag{1}$$

where $\nu \in (0,1)$, $f \in C^m[0,1]$, $a, b \in C^{2m}([0,1]\times[0,1])$, $m \in \mathbb{N}$, and the corresponding homogeneous equation has only the trivial solution. By a *fast* (C, C^m) *solver* of (1) we mean a solver which produces approximate solutions u_n, $n \in \mathbb{N}$, such that

- given the values of a, b and f at certain not more than $n_\star$ points depending on the solver (with $n_\star \to \infty$ as $n \to \infty$), the parameters of u_n can be determined at the cost of $\gamma_m n_\star$ arithmetical operations, and an accuracy

$$\| u - u_n \|_{C[0,1]} \le c_m n_\star^{-m} \| f \|_{C^m[0,1]} \tag{2}$$

  is achieved where u is the solution of (1);
- having the parameters of u_n in hand, the value of u_n at any point $x \in [0,1]$ is available at the cost of γ'_m operations.

Here the constants c_m, γ_m, γ'_m are independent of f and n. It occurs that estimate (2) is information optimal – in the worst case, under above assumptions, a higher order of error estimate cannot be achieved allowing more arithmetical work.

In a *fast* (L^p, C^m) *solver*, $\| u - u_n \|_{L^p(0,1)} \le c_m n_\star^{-m} \| f \|_{C^m[0,1]}$ is required instead of (2). In a *quasifast* (C, C^m) *solver,* $\| u - u_n \|_{C[0,1]} \le c_m n_\star^{-m} \log n_\star \| f \|_{C^m[0,1]}$ is required instead of (2).

In the literature, fast (C, C^m) solvers have been constructed only for integral equations without singularities that for (1) corresponds to the case $a \equiv 0$. We consider (1) in general case and construct a solver which is (C, C^m) quasifast and (L^p, C^m) fast for $1 \le p < \infty$; under slightly strengthened smoothness assumptions that the mth derivatives of a and b are Hoelder continuous, this solver is also (C, C^m) fast. Hence the complexity of (1) is the same as that for integral equations with smooth kernels or close to it. Actually some boundary singularities of $a, b \in C^{2m}([0,1]\times(0,1))$ and $f \in C[0,1] \cap C^m(0,1)$ are allowed in the final formulations.