On the integral equation formulations of some 2D contact problems.

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We consider integral equation formulations of contact problems of Kirchhoff (thin) plates, discontinuously supported along arcs or segments.

In the classical PDE formulation (biharmonic equation), the plate deflection is assumed as the primary unknown. The boundary conditions along the supported edge are imposed in terms of the plate deflection and its derivatives, and the reaction forces and couples (or moments) are subsequently computed from the deflection field.

To derive an equivalent integral equation representation, a preventative selection of suitable types of (equivalent) reaction forces and couples is needed, and the plate deflection is formulated as integral transformations (based upon suitable Green functions) of the unknown reaction forces. Each reaction component, through the associated integral transform having a specific kernel, defines a corresponding plate deflection component. In particular the shear force and the twisting moment have different integral transform kernels: the second one is the derivative of the first one. Therefore, to consider a reaction composed only by a shear force, or by a twisting moment, or by the sum of the two, in general is not irrelevant.

If the contact reactions are correctly chosen, then the integral representation of the plate deflection satisfies the biharmonic equation as well as all the contact problem boundary conditions, except that expressing the requirement that the plate deformed shape matches with the support profile. This compatibility condition is then reduced to an integral equation whose solution, if it exists, defines the unknown reaction (see [2], [3]).

Therefore the key question is the existence, uniqueness and endpoint behavior of the solution of this equation. Whenever the solution is uniquely defined in a proper space, hence determined, the plate deflection can be computed everywhere by means of its integral representation.

To describe the type of difficulties one may encounter when dealing with these problems, as an example we will consider the simple case of an infinite plate resting on a central segment. If the reaction is assumed to be described by a distributed shear force $q(y)$, the final integral equation has the form

$$(1) \qquad fp\int_{-1}^{1}[(x-y)^2\ln|x-y|+\alpha(x-y)+\beta]q(y)dy=f(x) \qquad -1<x<1$$

where fp means: finite-part value, $f(x)$ is a (smooth) known function, and α and β are constants to be properly determined.

If the reaction is assumed to be described by a distributed twisting moment $m(y)$, then we have

$$(2) \qquad fp\int_{-1}^{1}[(x-y)(2\ln|x-y|+1)+\alpha]m(y)dy=f(x) \qquad -1<x<1$$

where α is a constant to be determined.

The representation of the (weakly singular) solution of the equation

$$(3) \qquad \int_{-1}^{1}[2\ln|x-y|+3]g(y)dy=f(x), \qquad -1<x<1$$

which can be easily obtained from [1], is the starting point for the study of equations (1) and (2); these will be examined in spaces of functions having endpoint hypersingularities.

After having established a relationship between the integral operators defining the above equations, we will discuss the existence and uniqueness of solutions for (1) and (2) and examine their hypersingular endpoint behavior.

The numerical solution of these equations, in particular the first two, will also be briefly discussed.

References

[1] R. Estrada, R. P. Kanwal, *Singular Integral Equations*, Birkhäuser, Boston, 2000.

[2] G. Monegato, A. Strozzi, On the Form of the Contact Reaction in a Solid Circular Plate simply Supported along two Antipodal Edge Arcs and Deected by a Central Transverse Concentrated Force. J. Elasticity, 68, 2002, 13-35.

[3] A. Strozzi,G. Monegato, On the incompatibility between the equivalent shear force concept and the integral formulation of contact problems between Kirchhoff plates and irregular linear supports. Proc. Instns Mech Engrs Part C: Journal of Mechanical Engineering Science, v. 222, 2008, 1149-1163.