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SINGULAR POSITIVE SOLUTIONS TO A PROBLEM ARISING FROM HEMORHEOLOGY

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Whole blood is a concentrated suspension of cellular elements like red blood cells (RBCs), white blood cells and platelets, in an aqueous ionic solution called plasma. While plasma can be considered a Newtonian fluid, whole blood can have remarkable non-Newtonian properties (see [1] and references therein) like shear-thinning. At low shear rates RBCs aggregate in large structures called *rouleaux*, offering an increased resistance to motion and thus presenting a high apparent viscosity. As shear rate increases, the *rouleaux* break up and the individual cells align with the main flow direction, decreasing the viscosity. The basic equations describing the isothermal flow of an incompressible fluid in a domain $\Omega \in \mathbb{R}^3$ follow from the conservation of linear momentum and mass,

$$\rho \left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} \right) = -\nabla p + \operatorname{div}(\boldsymbol{\tau}), \quad \operatorname{div}(\boldsymbol{u}) = 0 \tag{1}$$

Here $(\boldsymbol{u}, p, \boldsymbol{\tau})$ are the unknown velocity, pressure and extra-stress tensor and ρ is the density of the fluid. In the case of generalized Newtonian fluids, $\boldsymbol{\tau}$ is a nonlinear function of the symmetric part of the velocity gradient $\boldsymbol{D} = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$, given by $\boldsymbol{\tau} = \mu(\dot{\gamma})\boldsymbol{D}$, where the shear rate $\dot{\gamma}$ is defined by $\sqrt{\frac{1}{2}\boldsymbol{D} \otimes \boldsymbol{D}}$. One of the most popular viscosity models, coming from the study of polymers, is the power-law model, where the viscosity, given by $\mu = K\dot{\gamma}^{n-1}$, has a singularity in the origin (in the case of pseudo-plastic or shear-thinning fluids, i.e $n < 1$). The unboundness of this viscosity function is normally considered unphysical and bounded power-law models, like Carreau-Yasuda or Cross, are used instead. However this is not mandatory and under some flow conditions it is possible to prove the existence of regular, bounded solutions. In this short presentation we show that the reverse is also true. Referring to [2], we show that for some scalar simplified version of (1) it is possible to show the existence of positive singular solutions for the whole space problem

$$\begin{cases} \operatorname{div}(|\dot{\gamma}|^{n-1}\nabla u) + f(u) = 0, \\ \lim\limits_{x\to\infty} u(x) = 0, \quad \lim\limits_{x\to 0} u(x) = \infty \end{cases} \tag{2}$$

The question remains whether singular solutions can exist for the initial set of equations, in general domains, and if they can occur for physically relevant values of the parameters.

REFERENCES

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[2] J. Zhou, Z. Yang and J. Zhao. Existence of singular positive solutions for a class of quasilinear elliptic equations. *Applied Mathematics and Computation*, **190** 2007, 423 – 431.