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MR2312350 (2008b:13032) 13F20

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The order of algebras with nontrivial fixed point subalgebras. (English summary)

Lobachevskii J. Math. **25** (2007), 187–196 (*electronic*).

This paper concerns algebras of the form $A = \mathbb{R}[x_1, \ldots, x_n]/\mathfrak{i}$, where $\mathfrak{m}^{r+1} \subset \mathfrak{i} \subset \mathfrak{m}^2$ for some positive integer r, and $\mathfrak{m} = \langle x_1, \ldots, x_n \rangle$ is a maximal ideal in the real polynomial ring in r indeterminates. In such an algebra, called a Weil algebra, the nilpotent elements form an ideal $\mathfrak{n}$. The width of A, denoted by $w(A)$, is defined to be $\dim\left(\mathfrak{n}/\mathfrak{n}^2\right)$. The order of A, denoted by $o(A)$, is the minimal r such that $\mathfrak{n}^{r+1} = 0$. In the case where $\mathfrak{i} = \mathfrak{m}^{r+1}$ then $w(A) = n$ and $o(A) = r$. Any real commutative local algebra with identity having both $\dim(\mathfrak{n}) < \infty$ and $A/\mathfrak{n} = \mathbb{R}$ is isomorphic to an algebra of the form considered above. Let SA be the subalgebra of elements fixed by every element in $\mathrm{Aut}(A)$. Evidently, $\mathbb{R} \subset SA$ since algebra automorphisms fix 1. If $SA = \mathbb{R}$, then A is said to have a trivial fixed-point subalgebra.

The main result of this paper states (i) there do not exist Weil algebras A with $w(A) = 1$ and $SA \neq \mathbb{R}$, (ii) there exist Weil algebras A with $w(A) = 2$ and $SA \neq \mathbb{R}$ if and only if $o(A) \geq 4$, and (iii) there exist Weil algebras A for any $w(A) \geq 3$ having $SA \neq \mathbb{R}$ if and only if $o(A) \geq 3$. The proof is based on results in [M. Kureš and W. M. Mikulski, Irish Math. Soc. Bull. No. 49 (2002), 23–41; MR1960041 (2003j:58004); Czechoslovak Math. J. **54(129)** (2004), no. 4, 855–867; MR2099999 (2006a:58003)] and on computations, described in this paper, showing the existence of Weil algebras with nontrivial SA.

Reviewed by *Sean Lawton*

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