Instituto Superior Técnico
Departamento de Matemática
Secção de Álgebra e Análise

Riemannian Geometry

Homework 13

due on Wednesday, June 14

1. *(Gauss–Bonnet Theorem for manifolds with boundary)* Let M be a compact, oriented, 2-dimensional manifold with boundary and let X be a vector field in M transverse to ∂M (i.e. such that $X_p \notin T_p\partial M$ for all $p \in \partial M$), with isolated singularities $p_1, \ldots, p_k \in M \backslash \partial M$. Prove that

$$\int_M K + \int_{\partial M} k_g = 2\pi \sum_{i=1}^{k} I_{p_i}$$

for any Riemannian metric on M, where K is the Gauss curvature of M and k_g is the geodesic curvature of ∂M.

2. Let (M, g) be a 2-dimensional manifold, with negative Gauss curvature. Show that a closed geodesic in M cannot be homotopic to a point.