

Symplectic Geometry

2nd Semester 2012/2013

Symplectic geometry studies the geometry of manifolds equipped with a non-degenerate, closed 2-form. Symplectic structures have their origin in the study of classical mechanics. Important developments in the early 1980's led to the emergence of new questions about these structures. Since then the subject has grown into one of the most active and vibrant areas of mathematical research. A distinguished feature of symplectic geometry has always been its close interaction with other fields: complex and algebraic geometry, mathematical physics and low dimensional topology. This course is intended for students enrolled in the Master or PhD Mathematics Program with knowledge on Differential Geometry and Algebraic Topology. Its main goal is to understand the foundations of symplectic geometry as well as to study some recent problems in this field.

1. Introduction to symplectic manifolds

- Symplectic linear algebra and symplectic manifolds.
- Symplectomorphisms, symplectic and Hamiltonian vector fields. Cotangent bundle.

2. Darboux-Moser-Weinstein Theory

- Isotopies. Classical Darboux Theorem. Moser Local Theorem.
- Weinstein Lagrangian Neighborhood Theorem.

3. Almost complex structures

- Compatible almost complex structures.
- Integrability. Kähler manifolds.

4. Hamiltonian actions

- Moment maps.
- Symplectic reduction: Marsden-Meyer-Weinstein Theorem.
- Toric manifolds.
- Convexity of the moment map: Atiyah-Guillemin-Sternberg Theorem.

5. J -holomorphic curves

- Gromov Compactness Theorem.
- Gromov Nonsqueezing Theorem.
- Applications to symplectic topology: properties in dimension 4.

Bibliography

1. A. Cannas da Silva, *Lectures on Symplectic Geometry*, Lecture Notes in Mathematics, **1764**, Springer-Verlag (2001).
2. K. Cieliebak, Introduction to Symplectic Geometry, Part A and Part B (Ver página).
3. D. McDuff and D. Salamon, *Introduction to Symplectic Topology*, Oxford Mathematical Monographs, Oxford University Press, New York (1995).
4. D. McDuff and D. Salamon, *J -holomorphic curves and symplectic topology*, American Mathematical Society Colloquium Publications, **52**. American Mathematical Society (2012).

Evaluation

Homework.

Lecturer: **Sílvia Anjos** <sanjos@math.ist.utl.pt>

Web Page: <http://www.math.ist.utl.pt/~sanjos/GS>