Instituto Superior Técnico
Departamento de Matemática
Secção de Álgebra e Análise

Differential Geometry

Homework 8

due on Wednesday, November 16

1. Let M be a compact and oriented smooth manifold with boundary $\partial M \neq \emptyset$. Show that there is no smooth map $\Phi : M \to \partial M$ such that $\Phi_{|\partial M} = \mathrm{id}$.
2. Prove the **Brouwer's fixed-point theorem**: any smooth map $\Psi : B \to B$ from the closed ball $B = \{x \in \mathbb{R}^n : |x| \leq 1\}$ into itself has a fixed point, that is, a point $p \in B$ such that $\Psi(p) = p$.