Instituto Superior Técnico
Departamento de Matemática
Secção de Álgebra e Análise

Differential Geometry

Homework 6

due on Tuesday, October 30

1. Let G be a Lie group.
   (a) Show that if X is a left invariant vector field then the flow of X satisfies $\phi_X^t = R_{\exp(tX)}$, where R denotes right multiplication.
   (b) Show that if G is an abelian Lie group then its Lie algebra is abelian (i.e., $[X,Y] = 0\ \forall X, Y \in \mathfrak{g}$).

2. Let $G_k(\mathbb{R}^d)$ denote the set of all linear subspaces of $\mathbb{R}^d$ of dimension k.
   (a) Show tat there is a bijection between $G_k(\mathbb{R}^d)$ and the quotient $O(d)/O(k)\times O(d-k)$. Hence $G_k(\mathbb{R}^d)$has a smooth structure for which this bijection becomes a diffeomorphism. This gives $G_k(\mathbb{R}^d)$ the structure of a manifold called the **Grassmanian manifold** of k-planes in $\mathbb{R}^d$; note that $G_1(\mathbb{R}^{d+1}) = \mathbb{RP}^d$.
   (b) Show that $\dim G_k(\mathbb{R}^d) = k(d-k)$.