

Differential Geometry

Homework 4

due on Tuesday, October 17

1. Let $\Phi : M \rightarrow N$ be a smooth map. Show that if X and $Y \in \mathfrak{X}(M)$ are Φ -related with Z and $W \in \mathfrak{X}(N)$, respectively, then $[X, Y]$ is Φ -related with $[Z, W]$.
2. Let $X, Y \in \mathfrak{X}(M)$ be complete vector fields with flows ϕ_X^t and ϕ_Y^s . Show that:
 - (a) given a diffeomorphism $\Phi : M \rightarrow M$, we have $\Phi_* X = X$ iff $\Phi \circ \phi_X^t = \phi_X^t \circ \Phi \forall t$.
 - (b) $\phi_Y^s \circ \phi_X^t = \phi_X^t \circ \phi_Y^s$ for all t and s iff $[X, Y] = 0$.
3. Consider the distribution D in \mathbb{R}^3 generated by the vector fields:

$$\frac{\partial}{\partial x} + \cos x \cos y \frac{\partial}{\partial z}, \quad \frac{\partial}{\partial y} - \sin x \sin y \frac{\partial}{\partial z}.$$

Show that D is involutive and determine the foliation \mathcal{F} that integrates it.