

Differential Geometry

Homework 4

due on Wednesday, October 19

1. Find a vector field X in \mathbb{R}^3 which is not complete.
2. Let $X, Y \in \mathfrak{X}(M)$ be complete vector fields with flows ϕ_X^t and ϕ_Y^s . Show that:
 - (a) given a diffeomorphism $\Phi : M \rightarrow M$, we have $\Phi_*X = X$ iff $\Phi \circ \phi_X^t = \phi_X^t \circ \Phi \forall t$.
 - (b) $\phi_Y^s \circ \phi_X^t = \phi_X^t \circ \phi_Y^s$ for all t and s iff $[X, Y] = 0$.
3. Consider the distribution D in \mathbb{R}^3 generated by the vector fields:

$$\frac{\partial}{\partial x} + \cos x \cos y \frac{\partial}{\partial z}, \quad \frac{\partial}{\partial y} - \sin x \sin y \frac{\partial}{\partial z}.$$

Show that D is involutive and determine the foliation \mathcal{F} that integrates it.