Instituto Superior Técnico
Departamento de Matemática
Secção de Álgebra e Análise

Differential Geometry

Homework 3

due on Tuesday, October 9

1. Let $\Phi : M \to N$ be a smooth map. Show that its graph

$$\mathcal{G}(\Phi) = \{(p, \Phi(p)) : p \in M\} \subset M \times N,$$

   is a proper submanifold of $M \times N$.

2. Let $\Psi : G \to \mathrm{Diff}(M)$ be a free action.

   (a) Show that if G is a finite group then Ψ is properly discontinuous.

   (b) Show that if Ψ is properly discontinuous then G is countable.

   (c) Give an example of an action, by a countable group, which is not properly discontinuous.

3. Find a vector field X in $\mathbb{R}$ which is not complete.