Instituto Superior Técnico
Departamento de Matemática
Secção de Álgebra e Análise

Differential Geometry

Homework 3

due on Wednesday, October 12

1. Let $\Psi : G \to \mathrm{Diff}(M)$ be a free action.
   (a) Show that if G is a finite group then Ψ is properly discontinuous.
   (b) Show that if Ψ is properly discontinuous then G is countable.
   (c) Give an example of an action, by a countable group, which is not properly discontinuous.
2. For each $n \in \mathbb{N}$ determine a smooth manifold whose fundamental group is $\mathbb{Z}_n$.
   Hint: For each $n \in \mathbb{N}$ define an action $\Psi_n : \mathbb{Z}_n \to \mathrm{Diff}(S)$ where $S = \{(z, w) \in \mathbb{C}^2 : |z|^2 + |w|^2 = 1\}$ is diffeomorphic to S^3.