

# Differential Geometry

## Homework 13

*due on Wednesday, January 4*

Consider the complex line bundle  $\xi = (\pi, \mathbb{C}\mathbb{P}^2 \setminus \{[0, 0, 1], \mathbb{C}\mathbb{P}^1\})$  defined in Homework 12. Compute the following Chern classes:

1.  $c_1(\xi)$ ;
2.  $c_1(\xi^*)$ ;
3.  $c_1(\xi \otimes \xi^*)$ ;
4.  $c_1(\otimes^n \xi)$ ;
5.  $c_1(\otimes^n \xi^*)$ ;
6.  $c_1(T\mathbb{S}^2)$ .

**(Remark:** We see then that  $\{\otimes^n \xi\}_{n \in \mathbb{N}} \cup \{\xi \otimes \xi^*\} \cup \{\otimes^n \xi^*\}_{n \in \mathbb{N}}$  is a countable family of non-equivalent complex line bundles over  $\mathbb{S}^2$ ; in fact, one can show that this family contains **all** line complex bundles over  $\mathbb{S}^2$ .)