Instituto Superior Técnico
Departamento de Matemática
Secção de Álgebra e Análise

Differential Geometry

Homework 12

due on Tuesday, December 11

1. Show that the pull-back of the canonical line bundle γ_d^1 over $\mathbb{P}^d$ by the map $\psi : \mathbb{P}^1 \to \mathbb{P}^d$ given by
$$\psi\left([x^0, x^1]\right) = \left[(x^0)^d, (x^0)^{d-1}x^1, \ldots, x^0(x^1)^{d-1}, (x^1)^d\right]$$
is isomorphic to $\gamma_1^1 \otimes \cdots \otimes \gamma_1^1$ (tensor product with d terms).
(**Hint:** Consider the line bundle $\xi = (\pi, E, \mathbb{P}^d)$ defined in Homework 11).

2. Let $\xi = (\pi, E, M)$ be a vector bundle of rank r equipped with a connection ∇. Let $\{s_1, \ldots, s_r\}$ and $\{s'_1, \ldots, s'_r\}$ be two local frames in a trivializing open set $U \subset M$. Denote by $A = (a_i^j) \in C^\infty(U, \mathrm{GL}(r))$ the matrix of change of basis so that $s'_i = \sum_j a_i^j s_j$. Show that the matrices ω and ω' of the corresponding connection 1-forms are related by
$$\omega' = A^{-1}\omega A + A^{-1}dA.$$