Instituto Superior Técnico
Departamento de Matemática
Secção de Álgebra e Análise

Differential Geometry

Homework 11

due on Tuesday, December 4

Let $E = \mathbb{P}^{d+1} \backslash \{[0, \ldots, 0, 1]\}$, and $\pi : E \to \mathbb{P}^d$ the map given by

$$\pi\left(\left[x^0, \ldots, x^{d+1}\right]\right) = \left[x^0, \ldots, x^d\right].$$

Let $U_i = \{[x^0, \ldots, x^d] \in \mathbb{P}^d : x^{i-1} \neq 0\}$, $1 \leq i \leq d+1$, and $\phi_i : \pi^{-1}(U_i) \to U_i \times \mathbb{R}$ the maps given by

$$\phi_i\left(\left[u^1, \ldots, u^{i-1}, 1, u^i, \ldots, u^d, t\right]\right) = \left(\left[u^1, \ldots, u^{i-1}, 1, u^i, \ldots, u^d\right], t\right).$$

Show that:

1. (U_i, ϕ_i) , where $1 \leq i \leq d+1$, are trivialization charts and these define a line bundle $\xi = (\pi, E, \mathbb{P}^d)$.
2. ξ is isomorphic to the canonical line bundle γ_d^1 over $\mathbb{P}^d$.
3. γ_d^1 is not the trivial bundle. (**Hint:** Suppose there was an isomorphism Ψ between the trivial bundle and ξ, and compute the image by Ψ of the complement of the zero section of the trivial bundle).