

# Differential Geometry

## Homework 11

*due on Tuesday, December 5*

Let  $E = \mathbb{P}^{d+1} \setminus \{[0, \dots, 0, 1]\}$ , and  $\pi : E \rightarrow \mathbb{P}^d$  the map given by

$$\pi \left( [x^0, \dots, x^{d+1}] \right) = [x^0, \dots, x^d]$$

Show that:

1. The trivializations  $\phi^{-1} : \mathbb{R}^d \times \mathbb{R} \rightarrow E$  given by

$$\phi^{-1} \left( u^1, \dots, u^d, t \right) = [u^1, \dots, u^{i-1}, 1, u^i, \dots, u^d, t]$$

define a line bundle  $\xi = (\pi, E, \mathbb{P}^d)$ .

2.  $\xi$  is isomorphic to the canonical line bundle  $\gamma_d^1$  over  $\mathbb{P}^d$ .
3.  $\gamma_d^1$  is not the trivial bundle. (**Hint:** Note that the trivial bundle is orientable).
4. The pull-back of  $\gamma_d^1$  by the map  $\psi : \mathbb{P}^1 \rightarrow \mathbb{P}^d$  given by

$$\psi \left( [x^0, x^1] \right) = \left[ (x^0)^d, (x^0)^{d-1}x^1, \dots, x^0(x^1)^{d-1}, (x^1)^d \right]$$

is isomorphic to  $\gamma_1^1 \otimes \dots \otimes \gamma_1^1$  (tensor product with  $d$  terms).