Instituto Superior Técnico
Departamento de Matemática
Secção de Álgebra e Análise

Differential Geometry

Homework 11

due on Wednesday, December 7

Let $E = \mathbb{P}^{d+1} \backslash \{[0, \ldots, 0, 1]\}$, and $\pi : E \to \mathbb{P}^d$ the map given by

$$\pi\left(\left[x^0, \ldots, x^{d+1}\right]\right) = \left[x^0, \ldots, x^d\right]$$

.

Show that:

1. The trivializations $\phi^{-1} : \mathbb{R}^d \times \mathbb{R} \to E$ given by

$$\phi^{-1}\left(u^1, \ldots, u^d, t\right) = \left[u^1, \ldots, u^{i-1}, 1, u^i, \ldots, u^d, t\right]$$

   define a line bundle $\xi = (\pi, E, \mathbb{P}^d)$.

2. ξ is isomorphic to the canonical line bundle γ^1_{d+1} over $\mathbb{P}^d$.
3. γ^1_{d+1} is not the trivial bundle. (**Hint:** Note that the trivial bundle is orientable).
4. The pull-back of γ^1_{d+1} by the map $\psi : \mathbb{P}^1 \to \mathbb{P}^d$ given by

$$\psi\left(\left[x^0, x^1\right]\right) = \left[(x^0)^d, (x^0)^{d-1}x^1, \ldots, x^0(x^1)^{d-1}, (x^1)^d\right]$$

   is isomorphic to $\gamma^1_2 \otimes \cdots \otimes \gamma^1_2$ (tensor product with d terms).