Instituto Superior Técnico
Departamento de Matemática
Secção de Álgebra e Análise

Differential Geometry

Homework 10

due on Wednesday, November 30

1. Let $\Phi : \mathbb{S}^d \to \mathbb{S}^d$ be a diffeomorphism. Show that:
   (a) If Φ has no fixed points then Φ is homotopic to the antipodal map;
   (b) If d is even and Φ preserves orientations then Φ has at least one fixed point.
2. Consider the vector field $X \in \mathbb{S}^{2d-1}$ obtained by restriction of the vector field in $\mathbb{R}^{2d}$:
$$X = x^2 \frac{\partial}{\partial x^1} - x^1 \frac{\partial}{\partial x^2} + \ldots + x^{2d} \frac{\partial}{\partial x^{2d-1}} - x^{2d-1} \frac{\partial}{\partial x^{2d}}.$$
   Show that X induces a vector field $\overline{X}$ in $\mathbb{P}^{2d-1}$. Use this vector field to compute $\chi(\mathbb{P}^{2d-1})$.
3. Show that a vector bundle is trivial if and only if it admits a global frame.