Instituto Superior Técnico
Departamento de Matemática
Secção de Álgebra e Análise

Differential Geometry

Homework 1

due on Wednesday, September 28

1. Let (x, y, z) be the Cartesian coordinates in $\mathbb{R}^3$ and consider the usual spherical coordinates (r, θ, φ) defined by

$$\begin{cases} x = r \sin\theta \cos\varphi \\ y = r \sin\theta \operatorname{sen}\varphi \\ z = r\cos\theta \end{cases}$$

   Compute:

   (a) The components of the tangent vectors to $\mathbb{R}^3$ $\frac{\partial}{\partial r}, \frac{\partial}{\partial \theta}, \frac{\partial}{\partial \varphi}$ in Cartesian coordinates.

   (b) The components of the tangent vectors to $\mathbb{R}^3$ $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ in spherical coordinates.

2. Let $\Phi : \mathbb{P}^2 \to \mathbb{R}^3$ be the map defined by

$$\Phi([x : y : z]) = \frac{1}{x^2 + y^2 + z^2}(yz, xz, xy).$$

   Show that Φ is smooth and show that it only fails to be an immersion at 6 points.

3. Consider a submanifold (N, Φ) of M with N compact. Show that (N, Φ) is an embedded submanifold.