1. HOMEWORK - 5

1- a) Determine all the possible dimensions of cyclic codes of length 13 over $\mathbb{F}_3$.

b) Verify that $p(x) = x^3 + 2x + 1$ is irreducible over $\mathbb{F}_3$ and use it to determine a primitive 13-th root of unity.

c) Use the previous results to justify that $g(x) = x^7 + 2x^6 + x^5 + 2x^4 + x^2 + 2$ is the generator polynomial of a $[13, 6, d]$ cyclic code with $d \geq 5$.

d) Encode by some form of systematic encoding the source message $(0, 1, 1, 2, 0, 1)$.

e) Decode by error trapping the received word

$$r = (1, 0, 1, 1, 2, 1, 0, 1, 2, 0, 1, 2, 2),$$

assuming the error has weight less or equal than 2.

f) Find if the code corrects all bursts with weight 3.

2- Let C be the binary $[15, 9]$ cyclic code with generator polynomial

$$g(x) = 1 + x^3 + x^4 + x^5 + x^6.$$

C is known to be a 3-burst error correcting code.
Decode $(0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 1, 1)$.

3- Let C be the $[15, 9, 7]$ cyclic code over $\mathbb{F}_{16} = \mathbb{F}_2[\alpha]$ $(\alpha^4 = \alpha + 1)$ with defining set $T = \{1, 2, 3, 4, 5, 6\}$.

a) Determine the corresponding generator polynomial.

b) Decode $r(x) = \alpha^7 x^{11} + \alpha^4 x^7 + \alpha^4 x^6 + \alpha^5 x^5 + \alpha^2 x^4 + x^3 + \alpha^{10} x^2 + \alpha^7$ using Peterson's algorithm.

4- Let A be the Reed-Solomon code over $\mathbb{F}_{3^2}$ defined by

$$C = \{(f(1), f(\lambda), \cdots, f(\lambda^7)) : f(x) \in P_4 \subset \mathbb{F}_{3^2}[x]\}$$

where λ is a primitive 8-root of unity satisfying $\lambda^2 = \lambda + 1$, and P_4 denotes as usual the vector space of polynomials with degree less than 4.

a) Determine the dimension and minimal distance of the trace code $Tr(A) \subset \mathbb{F}_3^8$.

b) Let B be the $[4, 2, 3]$ code over F_3 with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix}.$$

Decode the following output from the concatenated code $A[B]$, using syndrome decoding for B and Peterson's algorithm or the GCD algorithm for A:

(2001 2201 2110 1002 1001 1011 0010 1011).

c) Prove that the concatenated decoding algorithm used in b) corrects all errors with weight $w < 6$, but that there are errors with weight 6 that are not corrected. Prove also that the algorithm succefully decodes any burst with length $l < 8$.

5- Let C be the alternant binary code with parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 \end{bmatrix},$$

where $\alpha \in \mathbb{F}_8$ satisfies $\alpha^3 + \alpha + 1 = 0$.

a) Determine the dimension and minimal distance of C.

b) Apply the Euclidean algorithm to decode $1, 0, 1, 1, 0, 1, 1$.