

OSCILLATIONS OF DELAY AND DIFFERENCE EQUATIONS

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ABSTRACT

Consider the first-order linear retarded differential equation

$$x'(t) + p(t)x(\tau(t)) = 0, \quad t \geq t_0, \quad (1)$$

where $p, \tau \in C([t_0, \infty), \mathbb{R}^+)$, $\tau(t)$ is nondecreasing, $\tau(t) < t$ for $t \geq t_0$ and $\lim_{t \rightarrow \infty} \tau(t) = \infty$, and the (discrete analogue) retarded difference equation

$$\Delta x(n) + p(n)x(\tau(n)) = 0, \quad n = 0, 1, 2, \dots \quad (1)'$$

where $\Delta x(n) = x(n+1) - x(n)$, $p(n)$ is a sequence of nonnegative real numbers and $\tau(n)$ is a nondecreasing sequence of integers such that $\tau(n) \leq n-1$ for all $n \geq 0$ and $\lim_{n \rightarrow \infty} \tau(n) = \infty$. The state-of-the-art on the oscillation of all solutions to these equations is presented. Equations with several variable retarded or advanced arguments are also considered.

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