

1. Calcule caso exista:

(a)

$$\int_D \int \frac{dx dy}{\sqrt{xy}} \quad \text{onde } D = [0, 1] \times [0, 1]$$

(b)

$$\int_D \int \frac{dx dy}{\sqrt{|x-y|}} \quad \text{onde } D = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 1, y \leq x\}$$

(c)

$$\int_D \int \frac{y}{x} dx dy \quad \text{onde } D \text{ é delimitada por } x = 1, x = y, x = 2y$$

(d)

$$\int_0^1 \int_0^{e^v} \log x \, dx dy$$

(e) Discuta em função do parâmetro real λ :

$$\int_D \int \frac{dx dy}{(x^2 + y^2)^\lambda} \quad \text{onde } D \text{ é o disco unitário centrado na origem}$$

2. Calcule os seguintes integrais de caminho $\int_{\vec{c}} f \, ds$:

(a)

$$f(x, y, z) = x + y + z \quad \vec{c}: t \mapsto (\sin t, \cos t, t) \quad t \in [0, 2\pi]$$

(b)

$$f(x, y, z) = \cos z \quad \vec{c}: t \mapsto (\sin t, \cos t, t) \quad t \in [0, 2\pi]$$

(c)

$$f(x, y, z) = e^{\sqrt{z}} \quad \vec{c}: t \mapsto (1, 2, t^2) \quad t \in [0, 1]$$

(d)

$$f(x, y, z) = yz \quad \vec{c}: t \mapsto (t, 3t, 2t) \quad t \in [1, 3]$$

(e)

$$f(x, y, z) = x \cos z \quad \vec{c}: t \mapsto (t, t^2, 0) \quad t \in [0, 1]$$

(f)

$$f(x, y, z) = \frac{x+y}{y+z} \quad \vec{c}: t \mapsto (t, \frac{2}{3}t^{2/3}, t) \quad t \in [1, 2]$$