Real Analysis
MMAC
S1 – 2026/27

Instructors. Diogo Oliveira e Silva, Giuseppe Lamberti and Maciej Kucharski

Website (Fénix 2026/27).
`https://fenix.tecnico.ulisboa.pt/disciplinas/AR/2026-2027/1-semestre`

Prerequisites. Calculus and linear algebra. Students with a BSc in Physics or Engineering and an interest in analysis are welcome to enrol and take the course.

Goals. The course is divided into a preliminary part and a main part.

The preliminary part consists of a fast-paced but rigorous introduction to measure theory and integration, following Chapters 1–3 of Folland's *Real Analysis* book, and covering measure spaces, measurable functions, integration, product measures, signed measures, absolute continuity, the Radon–Nikodym theorem, and differentiation. **This part** will be taught by Giuseppe Lamberti and **is intended to make the course accessible to students who have not previously taken a full course in measure and integration**.

In the main part, real analysis is studied in a rigorous way, with emphasis on functional analysis, L^p spaces, Fourier analysis, and distributions. The course follows selected material from Chapters 5–6 and 8–9 of Folland's book; Chapter 4 on point set topology is assumed as prerequisite material and recalled as needed, while Radon measures (Chapter 7) will not be treated systematically. The emphasis is on the structural results and techniques that form the common language of modern analysis and that underlie many applications to functional analysis, harmonic analysis, partial differential equations, geometry, and probability. The course ends with an introduction to the Kakeya problem, including the two-dimensional theorem that Kakeya sets have Hausdorff dimension 2, and a discussion of the 2025 breakthrough of Wang–Zahl (leading to one of the 2026 Fields Medals) on the three-dimensional Kakeya conjecture.

Syllabus.

- Preliminaries. Measure and Integration.
- Elements of Funcional Analysis. Normed vector spaces, linear functionals, Hilbert spaces.
- L^p spaces. Basic theory, duality, distribution functions, weak L^p, interpolation.
- Elements of Fourier Analysis. Convolutions, Fourier transform, summation, pointwise convergence, applications.
- Elements of Distribution Theory. Compactly supported, tempered and periodic distributions, Sobolev spaces.

- Advanced Topic. The Kakeya Problem.

Bibliography.

- G. B. Folland, *Real analysis. Modern techniques and their applications.* Second edition. Pure Appl. Math. (N. Y.) Wiley-Intersci. Publ. John Wiley & Sons, Inc., New York, 1999.
- E. M. Stein, R. Shakarchi, *Real analysis. Measure theory, integration, and Hilbert spaces.* Princeton Lect. Anal., 3 Princeton University Press, Princeton, NJ, 2005.

Lectures.

– Monday 9:30AM–11:30AM Room V1.32 (Pav. Civil)
– Friday 3PM–4:30PM Room I9 (Pav. Minas)

Problem sessions.

– Thursday 12PM–1:30PM Room V0.09 (Pav. Civil)

Office hours. Held by prior appointment via email.

Course credit. Assessment in the RA course consists of one test (T60) lasting 60 minutes and worth 50%, and a final exam lasting 90 minutes and worth 50%. The resit exam has a duration of 120 minutes and a weight of 100%. Students with scores above 18 (out of 20) must take an oral examination.

Dates of the written exams:

- **T60: December 2026**, TBD
- **Exam: 8 January 2027**, Fr, TBD
- **Resit Exam: 25 January 2027**, Mo, TBD

Personal identification: Students may only present themselves for written exams with a Passport, National ID Card (e.g. "Cartão de Cidadão") or IST Student Card.

DM–IST, Av. Rovisco Pais, 1049-001 Lisboa, Portugal
Email address: diogo.oliveira.e.silva@tecnico.ulisboa.pt
Email address: giuseppe.lamberti@tecnico.ulisboa.pt
Email address: maciej.kucharski@tecnico.ulisboa.pt