Complex Analysis
MMAC
S1 – 2025/26

Instructor. Diogo Oliveira e Silva

Website (Fénix).
`https://fenix.tecnico.ulisboa.pt/disciplinas/CAC112/2025-2026/1-semestre`

Goals. The course is divided into a common core and three options to be taught in consecutive years. In the common core, complex analysis is studied in a rigorous way, together with some relatively immediate classical applications. The three options consist of more sophisticated applications of complex analysis. Option 1 concerns analytic number theory and culminates in the proof of the prime number theorem, proved by Hadamard and independently by de la Vallée Poussin in 1896, using ideas introduced by Riemann in 1859. Options 2 and 3 deal with the analysis of elliptic functions and theta functions, respectively.

Syllabus.

Common core.

- Preliminaries. Complex numbers and the complex plane, functions on the complex plane, integration along curves
- Cauchy's theorem: complements and applications. Morera's theorem, sequences of holomorphic functions, Schwarz reflection principle, Runge's approximation theorem
- Meromorphic functions and the logarithm. Zeros and poles, residue formula, singularities, argument principle, homotopies and simply connected domains, Fourier series and harmonic functions
- Fourier transform. Paley–Wiener theorem
- Entire functions. Jensen's formula, functions of finite order, infinite products, theorems of Weierstrass and Hadamard
- Conformal mappings. Conformal equivalence (examples: disk and upper half-plane), Schwarz lemma, Montel's theorem, Riemann mapping theorem

Option 1. (2025/26) The Zeta function and the prime number theorem
Option 2. (2026/27) Introduction to elliptic functions
Option 3. (2027/28) Applications of theta functions

Bibliography.

- E. M. Stein and R. Shakarchi, *Complex Analysis.* Princeton Lectures in Analysis, Princeton Univ. Press, Princeton, NJ, 2003.

Lectures.

– Tuesday 3:30PM–5:30PM Room 3.31 (Pav. Matemática)
– Friday 2:30PM–4PM Room 3.31 (Pav. Matemática)

Office hours. Held by prior appointment via email.

Course credit. Assessment in the CA course consists of two tests (T30), each lasting 30 minutes and worth 25%, and a final exam lasting 90 minutes and worth 50%. The resit exam has a duration of 120 minutes and a weight of 100%. Grades above 18 (out of 20) must be defended in an oral examination.

Dates of the written exams:

- **First T30: 14th October 2025**, Tu, 3:40PM–4:10PM
- **Second T30: 16th December 2025**, Tu, 3:40PM–4:10PM
- **Exam: 20th January 2026**, Tu, 10:30AM–12PM
- **Resit Exam: 3rd February 2026**, Tu, 10:30AM–12:30PM

Personal identification: Students may only present themselves for written exams with a National ID Card (e.g. "Cartão de Cidadão") or IST Student Card.

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