IST - Álgebra Linear - 1^o semestre 2019/2020

2^a ficha de exercícios para as aulas práticas

1. Verifique que:

(i) $\begin{bmatrix} 6 & -9 \\ 4 & -6 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix}^{1000} = I$ **(iii)** $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}^2 = -I$

(iv) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}^{222} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}^{220} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **(v)** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 4 \\ 6 & 11 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 6 & 11 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(vi) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ **(vii)** $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

(viii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \\ 3 & 0 & 3 \end{bmatrix}$ **(ix)** $3\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 0 & -2 \\ 1 & -1 \end{bmatrix}^2 = \begin{bmatrix} 4 & 2 \\ \frac{7}{2} & -\frac{5}{2} \end{bmatrix}$

(x) $\left(\begin{bmatrix} 4 \\ -2 \end{bmatrix}\begin{bmatrix} -\frac{1}{2} & 1 \end{bmatrix}\right)^T - 2\begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -\frac{1}{4} & 2 \\ -3 & \sqrt{2} \end{bmatrix}^T = \begin{bmatrix} -7 & -2\sqrt{2}-11 \\ 9 & 2\sqrt{2}+10 \end{bmatrix}$

(xi) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (se $ad - bc \neq 0$) **(xii)**

A 2^a coluna de $\begin{bmatrix} -1 & 1 & -1 \\ -1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 6 & 2 & 9 \\ -7 & 1 & 6 \\ 9 & 1 & -7 \end{bmatrix}$ é $\begin{bmatrix} -1 & 1 & -1 \\ -1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix}$

(xiii) As constantes a, b e c que definem a função $y = ax^2 + bx + c$ cujo gráfico passa pelos pontos $(x_1, y_1), (x_2, y_2)$ e (x_3, y_3) (de abcissas distintas entre si), constituem as variáveis do sistema linear cuja matriz aumentada é dada por:

$$\left[\begin{array}{ccc|c} x_1^2 & x_1 & 1 & y_1 \\ x_2^2 & x_2 & 1 & y_2 \\ x_3^2 & x_3 & 1 & y_3 \end{array}\right].$$

2. Determine (se existirem) as inversas das seguintes matrizes.

(i) $[1]$ **(ii)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **(iv)** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **(v)** $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ **(vi)** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(vii) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ **(viii)** $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ **(ix)** $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ **(x)** $\frac{1}{3}\begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{bmatrix}$

(xi) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 5 \end{bmatrix}$ **(xii)** $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ com $ad - bc \neq 0$ **(xiii)** $\begin{bmatrix} 0 & 0 & 0 & k_1 \\ 0 & 0 & k_2 & 0 \\ 0 & k_3 & 0 & 0 \\ k_4 & 0 & 0 & 0 \end{bmatrix}$, com $k_1 k_2 k_3 k_4 \neq 0$

3. Seja $A \in \mathcal{M}_{n\times n}(\mathbb{R})$ tal que $A^2 + 2A + 2I = \mathbf{0}$. Verifique que A é invertível e determine a sua inversa.

4. **a)** Determine a matriz $A \in \mathcal{M}_{2\times 2}(\mathbb{R})$ tal que $2(A - I) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A - \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}^T$.

   b) Determine a matriz $A \in \mathcal{M}_{3\times 3}(\mathbb{R})$ tal que $\left(\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} A^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} A$.

5. Sejam $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 2 & 0 & 2 & 0 \end{bmatrix}$ e $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$. Determine o conjunto solução do sistema linear homogéneo $Au = 0$ e resolva o sistema linear $Au = b$.

6. Sejam $A \in \mathcal{M}_{3\times 3}(\mathbb{R})$ e $b = (1, 2, 3)$. Sejam $(1, 2, 3)$ e $(4, 4, 4)$ duas soluções do sistema linear $Au = b$. Determine, justificando, uma solução de $Au = b$ distinta das anteriores.

7. Seja $A = \begin{bmatrix} 1 & 0 & \\ 0 & 0 & \\ 2 & 3 & \end{bmatrix} \in \mathcal{M}_{3\times 3}(\mathbb{R})$ uma matriz invertível cuja 3^a coluna não é conhecida. Determine, justificando, a 1^a coluna da matriz A^{-1}.

8. Seja A tal que $A = 2A^T$. Mostre que $A = \mathbf{0}$.