STABILITY OF BLOOD FLOW IN A MULTILAYERED VISCOELASTIC TUBE

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Animal organisms provide abundant examples of flows of biological liquids in the distensible tubes where fluid-structure interactions are very important for the flow stability. Flow-induced instability leads to reduction of the blood flow through the vessel, to deviation of the shear stress distribution at the wall from the normal conditions. Stability of the physiological flows in the vessels can be provided by special structure of their walls. Blood vessels can be considered as multilayered thick-walled tubes from nonlinear incompressible viscoelastic material. Material parameters of the different layers have been estimated in experiments with the intact vessels and wall segments [1-4]. Although there is an extensive literature on structure and dynamics of arterial wall, our understanding of wall mechanics is still incomplete.

In the present paper the results of theoretical study of the stability of the flow of incompressible Newtonian liquid in the multilayered tethered compliant tube are presented. The wall of the tube is consisted of three layers with thicknesses h_1, h_2, h_3. The Navier-Stokes equations for the liquid at the absence of the external forces and the governing equations for the layers are

$$\operatorname{div} \bar{v} = 0, \qquad \frac{\partial \bar{v}}{\partial t} + (\bar{v}\nabla)\bar{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \bar{v} \tag{1}$$

$$\operatorname{div} \bar{u}^j = 0, \qquad \rho_w^j \frac{\partial^2 \bar{u}^j}{\partial t^2} = -\nabla p^j + \operatorname{div} \bar{\sigma}^j \tag{2}$$

where $\bar{v}, \rho$ and *n* are velocity, density and kinematic viscosity of the fluid, p and p^j are hydrostatic pressure in the fluid and the layers, $\bar{u}^j$, ρ_w^j, $\bar{\sigma}^j$, $j = 1,2,3$ are the displacements, densities and stress tensors for the layers. The constitutive equations for the layers are presented by Voight model of the viscoelastic material as

$$\sigma_i^j = A_{ik}^j \varepsilon_k^j + \mu_w^j \frac{\partial}{\partial t} \varepsilon_i^j \tag{3}$$

where $\sigma^{jT} = (\sigma_{rr}, \sigma_{\varphi\varphi}, \sigma_{zz}, \sigma_{\varphi z}, \sigma_{rz}, \sigma_{r\varphi})$, $\varepsilon^{jT} = (\varepsilon_{rr}, \varepsilon_{\varphi\varphi}, \varepsilon_{zz}, \varepsilon_{\varphi z}, \varepsilon_{rz}, \varepsilon_{r\varphi})$, A_{ik}^j is the matrix of elasticity modulus, μ_w^j is viscosity, $\varepsilon_{ik}^j = (\nabla_i u_k^j + \nabla_k u_i^j)/2$. The materials of the layers are considered as transversely isotropic and the components of A_{ik}^j are defined in [3].

The boundary conditions are:

$$r = R \quad : \qquad v_i = \frac{\partial u_i^1}{\partial t}, \ \nu\rho V_{rz} = \sigma_{rz}^1, \ \nu\rho V_{r\varphi} = \sigma_{r\varphi}^1, \ -p + \nu\rho V_{rr} = \sigma_{rr}^1 \tag{4}$$

$$r = R + h_1 \quad : \qquad u_i^1 = u_i^2, \ \sigma_{rz}^1 = \sigma_{rz}^2, \ \sigma_{r\varphi}^1 = \sigma_{r\varphi}^2, \ \sigma_{rr}^1 = \sigma_{rr}^2 \tag{5}$$

$$r = R + h_1 + h_2 \quad : \qquad u_i^2 = u_i^3, \ \sigma_{rz}^2 = \sigma_{rz}^3, \ \sigma_{r\varphi}^2 = \sigma_{r\varphi}^3, \ \sigma_{rr}^2 = \sigma_{rr}^3 \tag{6}$$

$$r = R + H \quad : \qquad \sigma^3_{rz} = 0,\ \sigma^3_{r\varphi} = 0,\ \sigma^3_{rr} = 0,\ \bar{u}^3 = 0 \tag{7}$$

where $V_{ik} = (\nabla_i v_k + \nabla_k v_i)/2$, R, H and L are the inner radius, thickness and length of the tube ($R/L << 1, H/R << 1$). Solution of the system has been obtained as a superposition of small-amplitude axisymmetric perturbations and the Hagen-Poiseuille velocity field in the fluid and the displacement fields in the layers. The method and the numerical procedure are presented in [5].

Theoretical analysis and calculations have revealed multifarious influence of the Young's module E^{1-3}, shear module G^{1-3} and viscosities μ^j_w of the layers on the stability of the flow. Some results are presented in fig.1-2. The isotropic uniform single-layered tube has one unstable mode [5]. Increasing $G^{1,3}$ and G^2 exhibit a stabilizing and destabilizing effects on this mode accordingly. For high values $G^{1,3}$ within the physiological range the system becomes stable. A small increase of μ^1_w leads to increasing the amplification rate of the unstable mode whereas increasing μ^2_w stabilizes the system. For some values of μ^2_w the group velocity $V_g = 0$ that suggests the existence of an absolute instability [5]. The comparative study of the uniform single-layered and the three-layered tubes revealed significant influence of the mechanical parameters and relative thicknesses of the layers on stability of the system. As far as thickening and increasing rigidity of separate layers are proper for different pathologies (hypertension, atherosclerosis etc) the stability of the vessel can be changed by the variations of the material parameters and the flow rate can be essentially decreased. At certain stages of the pathological processes the variations can be considered as adaptive mechanism which can be controlled by the inner (mechanosensitive) layer of the blood vessels. The presented results can be used for understanding the dynamics of the vessel wall and stability of the blood flow in the vessels in normal state and at different pathologies as well as for construction and optimization of the multilayered coating for the technological applications.

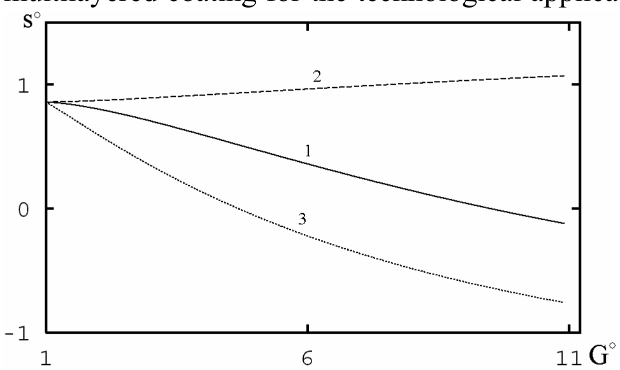

Fig.1. Amplification rate s° of the unstable mode versus the dimensionless shear modulus G° of one of the layers $j = 1,2,3$ (curves 1-3) while $G^\circ_k = 1$ for $k \neq j$.

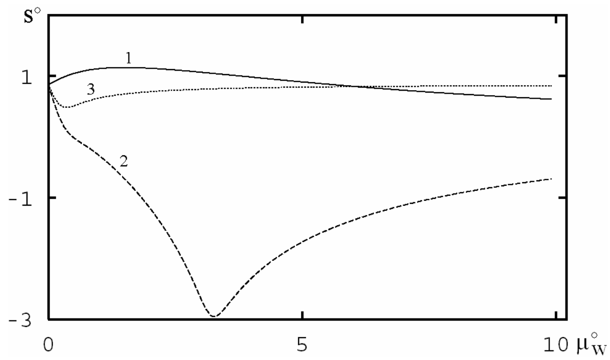

Fig.2. Amplification rate s° of the unstable mode versus the dimensionless viscosity μ°_w of one of the layers $j = 1,2,3$ (curves 1-3) while $\mu^\circ_{wk} = 0$ for $k \neq j$.

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