Quasiconvex functions incorporating an incompressibility constraint, and application to microstructure in smectic elastomers

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Smectic elastomers are layered materials exhibiting a solid-like elastic response along the layer normal and a rubbery one in the plane. The appropriate energy density W is transversally-isotropic, and infinity away from the unit-determinant surface. Experiment shows the formation of microstructure, which can be interpreted as a consequence of lack of quasiconvexity of W. One is therefore interested in determining the quasiconvex envelope of W.

We prove that a quasiconvex function $W : \mathbb{M}^{n\times n} \to [0,\infty]$ which is finite on the set $\Sigma = \{F : \det F = 1\}$ is rank-one convex, and hence continous, on Σ; and the same for constraints on minors. Hence the rank-one convex envelope gives an upper bound on the convex envelope. This result is based on an improvement of a construction by Müller and Šverák.

We then determine the relaxation of some transversally-isotropic energy densities with incompressibility constraints,. A special case is a model for smectic A elastomers. We discuss the implications of our result for the computation of macroscopic stress-strain curves, and compare to experiment. This part is based on joint work with Adams, Desimone, and Dolzmann.