

## Cálculo Diferencial e Integral 2 Respostas à Ficha de Trabalho 7

1. (a)  $\int_0^{\sqrt{2}} \int_{\frac{3\pi}{4}}^{\frac{9\pi}{4}} f(r \cos \theta, r \sin \theta) r d\theta dr.$   
 (b)  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_1^2 f(r \cos \theta, r \sin \theta) r dr d\theta.$   
 (c)  $\int_{-\frac{\pi}{4}}^0 \int_0^{\frac{1}{\cos \theta}} f(r \cos \theta, r \sin \theta) r dr d\theta + \int_0^{\frac{\pi}{2}} \int_0^1 f(r \cos \theta, r \sin \theta) r dr d\theta.$
2. (a)  $\pi \left(1 - \frac{1}{e^4}\right).$   
 (b)  $\frac{\pi \log 2}{2}.$   
 (c)  $\frac{3\pi}{4}.$   
 (d)  $\pi \sin(2).$   
 (e)  $\pi - 2 \arctan \frac{1}{2}.$
3. (a) A imagem de  $T$  é  $\{(x, y) \in \mathbb{R}^2: 0 \leq x \leq 2, -x^2 \leq y \leq x\}.$   
 (b)  $2 + \frac{2}{\sqrt{3}} \left( \arctan \left( \frac{1}{\sqrt{3}} \right) - \arctan \left( \frac{5}{\sqrt{3}} \right) \right).$
4.  $\frac{1}{16} \left(1 - \frac{1}{e}\right).$
5. (a)  $\int_0^{2\pi} \int_0^{\frac{1}{\sqrt{2}}} \int_{\rho}^{\sqrt{1-\rho^2}} f(\rho \cos \theta, \rho \sin \theta, z) \rho dz d\rho d\theta$  e  
 $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^1 f(r \cos \theta \sin \phi, r \sin \theta \sin \phi, r \cos \phi) r^2 \sin \phi dr d\phi d\theta.$   
 (b)  $\int_0^{\frac{\pi}{4}} \left( \int_0^1 \int_{\sqrt{1-\rho^2}}^{\sqrt{2-\rho^2}} f(\rho \cos \theta, \rho \sin \theta, z) \rho dz d\rho + \int_1^{\sqrt{2}} \int_0^{\sqrt{2-\rho^2}} f(\rho \cos \theta, \rho \sin \theta, z) \rho dz d\rho \right) d\theta$   
 e  $\int_0^{\frac{\pi}{4}} \int_0^{\frac{\pi}{2}} \int_1^{\sqrt{2}} f(r \cos \theta \sin \phi, r \sin \theta \sin \phi, r \cos \phi) r^2 \sin \phi dr d\phi d\theta.$
6. (a)  $\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_1^2 r^4 \sin^3 \phi dr d\phi d\theta$  e  
 $\int_0^{\frac{\pi}{2}} \left( \int_{-\frac{1}{\sqrt{2}}}^{-\frac{1}{\sqrt{2}}} \int_{-z}^{\sqrt{4-z^2}} \rho^3 d\rho dz + \int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \int_{\sqrt{1-z^2}}^{\sqrt{4-z^2}} \rho^3 d\rho dz + \int_{\frac{1}{\sqrt{2}}}^{\sqrt{2}} \int_z^{\sqrt{4-z^2}} \rho^3 d\rho dz \right) d\theta.$   
 (b)  $\frac{127 \cdot 43\pi}{420\sqrt{2}}.$
7.  $\frac{31}{20}.$
8. (a)  $\frac{2\pi}{3}.$   
 (b)  $4\pi^2.$
9. (a)  $-\frac{26}{3}.$   
 (b)  $\frac{\partial F}{\partial x}(x, y) = \int_0^1 \left( \int_0^u \frac{u}{1+(xu+y^2v)^2} dv \right) du; \quad \frac{\partial F}{\partial y}(x, y) = \int_0^1 \left( \int_0^u \frac{2yv}{1+(xu+y^2v)^2} dv \right) du.$   
 (c)  $G'(x) = -\sin x \log(1 + e^{x \cos x}) - 2x \log(1 + e^{x^3}) + \int_{x^2}^{\cos x} \frac{te^{tx}}{1+e^{tx}} dt.$
10.  $\frac{2\pi}{t} \left( 2e^{t^2} - e^t \right).$