# FORMULÁRIO

- **Interpolação Polinomial**

**Fórmula de Lagrange**:

$$l_j(x) = \prod_{i=0, i\neq j}^{n} \frac{x - x_i}{x_j - x_i}\,, \quad j = 0, 1, \ldots, n\,, \qquad\qquad p_n(x) = \sum_{j=0}^{n} f(x_j)\; l_j(x)$$

**Fórmula de Newton com diferenças divididas**:

$$\begin{cases} f[x_j] = f(x_j), \quad j = 0, ..., n \\ f[x_j, ..., x_{j+k}] = \dfrac{f[x_{j+1}, ..., x_{j+k}] - f[x_j, ..., x_{j+k-1}]}{x_{j+k} - x_j}, \quad j = 0, ..., n-k, \quad k = 1, ..., n \end{cases}$$

$$p_n(x) = f[x_0] + \sum_{i=1}^{n} f[x_0, ..., x_i](x - x_0)\cdots(x - x_{i-1})$$

**Fórmula de erro**: $e_n(x) = f(x) - p_n(x) = \dfrac{f^{(n+1)}(\xi)}{(n+1)!} \displaystyle\prod_{i=0}^{n}(x - x_i) = f[x_0, x_1, \ldots, x_n, x] \prod_{i=0}^{n}(x - x_i)$

- **Mínimos Quadrados**

$$\begin{bmatrix} (\phi_0, \phi_0) & \ldots & (\phi_0, \phi_N) \\ \ldots & \ldots & \ldots \\ (\phi_N, \phi_0) & \ldots & (\phi_N, \phi_N) \end{bmatrix} \begin{bmatrix} a_0 \\ \ldots \\ a_N \end{bmatrix} = \begin{bmatrix} (\phi_0, f) \\ \ldots \\ (\phi_N, f) \end{bmatrix}$$

$$(\phi_i, \phi_j) = \sum_{k=0}^{M} \phi_i(x_k)\phi_j(x_k), \qquad\qquad (\phi_i, f) = \sum_{k=0}^{M} \phi_i(x_k) f_k$$

- **Integração Numérica**

**Regra dos trapézios**:

$$Q_1(f) = \frac{b-a}{2}\Big[f(a) + f(b)\Big], \qquad Q_1^N(f) = \frac{h}{2}\left[f(x_0) + f(x_N) + 2\sum_{i=1}^{N-1} f(x_i)\right]$$

$$E_1^N(f) = -\frac{(b-a)\,h^2}{12} f''(\xi) \qquad\qquad \xi \in (a, b)$$

**Regra de Simpson**:

$$Q_2(f) = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right], \qquad Q_2^N(f) = \frac{h}{3}\left[f(x_0) + f(x_N) + 4\sum_{i=1}^{N/2} f(x_{2i-1}) + 2\sum_{i=1}^{N/2-1} f(x_{2i})\right]$$

$$E_2^N(f) = -\frac{(b-a)\,h^4}{180} f^{(4)}(\xi) \qquad\qquad \xi \in (a, b)$$

• **Métodos numéricos para equações diferenciais**

**Método de Euler explícito** $y_{i+1} = y_i + hf(t_i, y_i)$

$$|y(t_i) - y_i| \leq \frac{hM}{2L}\left[e^{L(t_i - t_0)} - 1\right], \qquad |y''(t)| \leq M,\ t \in [t_0, t_i]$$

**Método de Taylor de ordem 2**: $y_{i+1} = y_i + hf(t_i, y_i) + \dfrac{h^2}{2}\left(\dfrac{\partial f}{\partial t}(t_i, y_i) + \dfrac{\partial f}{\partial y}(t_i, y_i)\, f(t_i, y_i)\right)$

**Métodos de Runge-Kutta de ordem 2**: $y_{i+1} = y_i + \left(1 - \dfrac{1}{2\alpha}\right) hf(t_i, y_i) + \dfrac{1}{2\alpha} hf(t_i + \alpha h, y_i + \alpha h f(t_i, y_i))$

$\alpha = \frac{1}{2}$: Método de Euler modificado; $\alpha = 1$: Método de Heun

**Método de Runge-Kutta de ordem 4 clássico**: $y_{i+1} = y_i + \dfrac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$

$$k_1 = f(t_i, y_i), \quad k_2 = f(t_i + \tfrac{h}{2}, y_i + \tfrac{h}{2}k_1), \quad k_3 = f(t_i + \tfrac{h}{2}, y_i + \tfrac{h}{2}k_2), \quad k_4 = f(t_i + h, y_i + hk_3)$$

**Método de Euler implícito**: $y_{i+1} = y_i + hf(t_{i+1}, y_{i+1})$

**Método de Crank-Nicolson**: $y_{i+1} = y_i + \dfrac{h}{2}\left(f(t_i, y_i) + f(t_{i+1}, y_{i+1})\right)$