FORMULÁRIO

Teoria de erros e representação de números no computador

Erro absoluto e erro relativo: $(x, \tilde{x} \in \mathbb{R}, \quad x \approx \tilde{x})$

$$e_{\tilde{x}} = x - \tilde{x}, \qquad \delta_{\tilde{x}} = \frac{e_{\tilde{x}}}{x},\ x \neq 0$$

$$\text{erro absoluto} : |e_{\tilde{x}}|, \qquad \text{erro relativo} : |\delta_{\tilde{x}}|,\ x \neq 0$$

Erros de arredondamento: $(x = \sigma(0.a_1a_2...)_\beta \beta^t,\ a_1 \neq 0; \quad \tilde{x} = fl(x) \in FP(\beta, n, t_1, t_2))$

$$|e_{\tilde{x}}| \leq \beta^{t-n}, \quad |\delta_{\tilde{x}}| \leq \beta^{1-n} := U, \quad \text{(arredondamento por corte)}$$

$$|e_{\tilde{x}}| \leq \frac{1}{2}\beta^{t-n}, \quad |\delta_{\tilde{x}}| \leq \frac{1}{2}\beta^{1-n} := U, \quad \text{(arredondamento simétrico)}$$

Propagação de erros: $(x, \tilde{x} \in \mathbb{R}^n, \quad x \approx \tilde{x})$

$$e_{f(\tilde{x})} = f(x) - f(\tilde{x}) \approx \sum_{k=1}^{n} \frac{\partial f}{\partial x_k}(x) e_{\tilde{x}_k}$$

$$\delta_{f(\tilde{x})} = \frac{e_{f(\tilde{x})}}{f(x)} \approx \sum_{k=1}^{n} p_{f,k}(x)\delta_{\tilde{x}_k} \quad p_{f,k}(x) = \frac{x_k \frac{\partial f}{\partial x_k}(x)}{f(x)}$$

Métodos iterativos para equações não-lineares

Método da bissecção: $x_{k+1} = \dfrac{a_k + b_k}{2}, \quad f(a_k)f(b_k) < 0$

$$|x - x_{k+1}| \leq |x_{k+1} - x_k|, \qquad |x - x_k| \leq \frac{b-a}{2^k}$$

Método do ponto fixo: $x_{k+1} = g(x_k)$

$$|x - x_{k+1}| \leq \frac{L}{1-L}|x_{k+1} - x_k|,$$

$$|x - x_k| \leq L^k|x - x_0|, \qquad |x - x_k| \leq \frac{L^k}{1-L}|x_1 - x_0|$$

Método de Newton: $x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)}$

$$x - x_{k+1} = -\frac{f''(\xi_k)}{2f'(x_k)}(x - x_k)^2, \text{ com } \xi_k \text{ entre } z \text{ e } x_k$$

$$|x - x_k| \leq \frac{1}{\mathbb{K}}\left(\mathbb{K}|x - x_0|\right)^{2^k}$$

Método da secante: $x_{k+1} = x_k - f(x_k)\dfrac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}$

$$x - x_{k+1} = -\frac{f''(\xi_k)}{2f'(\eta_k)}(x - x_k)(x - x_{k-1}),\ \text{com } \xi_k, \eta_k \text{ num intervalo que contém } z, x_k \text{ e } x_{k-1}$$

$$|x - x_{k+1}| \leq \mathbb{K}\,|x - x_k|\,|x - x_{k-1}|\,, \quad \mathbb{K} = \frac{\max|f''|}{2\min|f'|}$$

Normas e Condicionamento

$$\|\mathbf{A}\|_\infty = \max_{1\leq i\leq n} \sum_{j=1}^{n} |a_{ij}| \qquad \text{cond}(\mathbf{A}) = \|\mathbf{A}\|\,\|\mathbf{A}^{-1}\|$$

$$\|\mathbf{A}\|_1 = \max_{1\leq j\leq n} \sum_{i=1}^{n} |a_{ij}| \qquad \|\delta_{\tilde{\mathbf{x}}}\| \leq \frac{\text{cond}(\mathbf{A})}{1 - \text{cond}(\mathbf{A})\,\|\delta_{\tilde{\mathbf{A}}}\|}\,(\|\delta_{\tilde{\mathbf{A}}}\| + \|\delta_{\tilde{\mathbf{b}}}\|),\ \text{sistema } \mathbf{Ax} = \mathbf{b}$$

$$\|\mathbf{A}\|_2 = (\rho(\mathbf{A}^T\mathbf{A}))^{1/2}$$

Métodos iterativos para sistemas lineares

$$\mathbf{Ax} = \mathbf{b} \Leftrightarrow \mathbf{x} = \mathbf{Cx} + \mathbf{d} \quad \rightarrow \quad \mathbf{x}^{(k+1)} = \mathbf{Cx}^{(k)} + \mathbf{d}$$

$$\|\mathbf{x} - \mathbf{x}^{(k)}\| \leq \|\mathbf{C}\|^k \|\mathbf{x} - \mathbf{x}^{(0)}\|, \qquad \|\mathbf{x} - \mathbf{x}^{(k)}\| \leq \frac{\|\mathbf{C}\|^k}{1 - \|\mathbf{C}\|}\|\mathbf{x}^{(1)} - \mathbf{x}^{(0)}\|$$

$$\|\mathbf{x} - \mathbf{x}^{(k+1)}\| \leq \frac{\|\mathbf{C}\|}{1 - \|\mathbf{C}\|}\|\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}\|$$

Método de Jacobi: $C = -\mathbf{D}^{-1}(\mathbf{L} + \mathbf{U})$ $\qquad x_i^{(k+1)} = (b_i - \sum_{j=1, j\neq i}^{n} a_{ij}\, x_j^{(k)})/a_{ii}$

Método de Gauss-Seidel:

$$C = -(\mathbf{L} + \mathbf{D})^{-1}\mathbf{U}$$
$$x_i^{(k+1)} = (b_i - \sum_{j=1}^{i-1} a_{ij}\, x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}\, x_j^{(k)})/a_{ii}$$

Método de Newton para sistemas não-lineares

$$\mathbf{J}(\mathbf{x}^{(k)})\Delta\mathbf{x}^{(k)} = -\mathbf{f}(\mathbf{x}^{(k)}) \qquad \mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta\mathbf{x}^{(k)}$$