COMBINATÓRIA E TEORIA DE CÓDIGOS
Homework 6

(deadline 6/6/2011)

Justify all your answers.

1. Let $C = \text{Ham}(3,2)$ be the binary Hamming code with redundancy 3 and generator polynomial $g(t) = 1 + t + t^3$.

   (a) Find the parameters and the generator polynomial of $C^{(3)}$.

   (b) Show that $C^{(3)}$ corrects all burst-m errors with $m \leq 3$.

   (c) Using the Burst Error Trapping Algorithm, decode the following received vector

   $$y(t) = t + t^3 + t^5 + t^7 + t^8 + t^9 + t^{11} .$$

2. Let C be the binary linear code with the following parity-check matrix

   $$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} .$$

   Find the minimum distance $d(C)$, and determine the code capacity for detecting and correcting random errors. Show also that C detects all burst-m errors with $m \leq 3$.

3. A q-ary cyclic code, with length n, is called *degenerate* if there is $r \in \mathbb{N}$ such that r divides n and each code word is of the form $c = c'c' \cdots c'$ with $c' \in \mathbb{F}_q^r$, i.e., each code word consists in n/r identical copies of a sequence c' with length r.

   (a) Let C be the code in the previous exercise. Show that the ponctured code, in the last coordinate, of the dual code $C^\perp$ is a degenerate cyclic code, and determine its generator polynomial.

   (b) Determine all degenerate, cyclic and binary codes with length 9, writing the generator polynomials and the corresponding r-sequences.

4. Let C be the Reed-Solomon code over $\mathbb{F}_8$ with generator polynomial $g(t) = (t-\alpha)(t-\alpha^2)(t-\alpha^3)$, where $\alpha \in \mathbb{F}_8$ is a root of $1 + t + t^3$.

   (a) Justifify that α is a primitive element in $\mathbb{F}_8$.

   (b) Find the parameters of C.

   (c) Find the parameters of the dual code $C^\perp$.

   (d) Find the parameters of of the extended code $\widehat{C}$.

   (e) Find the parameters of the concatenation code $C^* = \phi^*(C)$, where $\phi : \mathbb{F}_8 \to \mathbb{F}_2^3$ is the linear map defined by $\phi(1) = 100$, $\phi(\alpha) = 010$ and $\phi(\alpha^2) = 101$.

5. A linear code C is *self-orthognal* if $C \subseteq C^\perp$. Determine the generator polynomial of all self-orthogonal Reed-Solomon codes over $\mathbb{F}_{16}$. Which of these codes are self-dual?