COMBINATÓRIA E TEORIA DE CÓDIGOS

Homework 4 (deadline 26/4/2013, in class)

1. Let C be a binary, linear and self-dual code.

    (a) Show that, if $x, y \in C$ have weight a multiple of 4, the the weight of $x + y$ is also a multiple of 4.

    (b) Show that either the weight of all words in C is a multiple of 4, or the weight of half the code words is a multiple of 4 and weight of the other half is even but not divisible by 4.

    (c) Show that $\vec{1} = (1, \ldots, 1) \in C$.

    (d) If the lenght of code C is 6, determine the minimum distance $\mathrm{d}(C)$.

2. Without listing the code words, determine the number of words with weight 4 in the extended binary Hamming code $\widehat{\mathrm{Ham}}(3, 2)$.

3. For any code C we define the *weight enumerator polynomial*[1] by $W_C(t) = \sum_{i \geq 0} A_i t^i$, where

$$A_i = \#\{x \in C \, : \, \mathrm{w}(x) = i\} \ .$$

   Let $C \subset \mathbb{F}_2^8$ be a self-dual linear code. Determine all possible weight enumerator polynomials of C. Give an example of a self-dual code for each polynomial you found.

4. Let C be a binary perfect code with length n and minimum distance $2t + 1$. Show that there is a Steiner system $S(t + 2, 2t + 2, n + 1)$.

5. Determine the weight enumerator polynomial of the extended Golay code G_{24}.
[Sugestion: Show that $\vec{1} \in G_{24}$.]

6. Bonus exercise: continuing Exercise 3, show that, if C and C' are self-dual binary linear codes of length 8 with the same weight enumerator polynomial, then C and C' are equivalent codes.

---

[1] Note that the polynomial $W_C(t)$ is just the generating function for the sequence $\{A_i\}_{i \in \mathbb{N}_0}$