COMBINATÓRIA E TEORIA DE CÓDIGOS
HOMEWORK 4

(deadline 29/4/2011)

Justify all your answers.

1. Let C_1 e C_2 be q-ary linear codes with parameters $[n, k_1, d_1]$ and $[n, k_2, d_2]$, respectively.
   (a) Show that $C_1 * C_2$ (the Plotkin construction) is a linear code.
   (b) If G_1 and G_2 are generator matrices for C_1 and C_2, respectively, write a generator matrix for $C_1 * C_2$ in terms of G_1 and G_2.
   (c) If H_1 and H_2 are parity-check matrices for C_1 and C_2, respectively, write a parity-check matrix for $C_1 * C_2$ in terms of H_1 and H_2.

2. Let C be the binary code with the following parity-check matrix

$$H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

   (a) Determine the $[n, k, d]$ parameters of the code C.
   (b) Show that C can be used to correct all errors with weight 1 and all errors with weight 2 with a nonzero n-th component. Can this code correct simultaneously all these errors plus a few more with weight 2?
   (c) Describe a decoding algorithm that corrects all errors mentioned in part (b), and decode the received vector $y = 10111011$.

3. Let C be a q-ary MDS code with parameters $[n, k]$, where $k < n$.
   (a) Show that there is a q-ary MDS code with lenght n and dimention $n - k$.
   (b) Show that there is a q-ary MDS code with lenght $n - 1$ and dimention k.

4. Consider the vector space $V = \mathbb{F}_q^3$.
   (a) Show that V contains $\frac{q^3-1}{q-1} = q^2 + q + 1$ 1-dimentional vector subspaces.
   (b) Show that V contains $\frac{q^3-1}{q-1} = q^2 + q + 1$ 2-dimentional vector subespaces.
   (c) Let $\mathcal{P}$ be the set of 1-dimentional vector subspaces and let $\mathcal{B}$ be the set of 2-dimentional vector subspaces. Show that $\mathcal{P}$ (as the set of points) and $\mathcal{B}$ (as the set of blocks), with the relation $P \in \mathcal{P}$ belongs to $B \in \mathcal{B}$ if P is a subspace of B, define a Steiner system $S(2, q+1, q^2+q+1)$. Since the number of points and the number of blocks are the same, this Steiner system is called a 2-dimentional projective geometry (or a projective plane) of order q, and its denoted by $PG(2, q)$ or $PG_2(q)$.

5. For any code C, we define $A_i = \#\{x \in C : w(x) = i\}$. Determine the numbers A_i for the extended Golay code G_{24}. [Sugestion: Show that $\vec{1} \in G_{24}$.]