COMBINATÓRIA E TEORIA DE CÓDIGOS
HOMEWORK 3

(deadline 1/4/2011)

Justify all your answers.

1. Let C be a linear code with length $n \geq 4$. Let H be a parity-check matrix for C such that its columns are distinct and have odd weight. Show that $d(C) \geq 4$.

2. (a) For a q-ary linear code, with lenght n and minimum distance d, show that the vectors $x \in \mathbb{F}_q^n$ with weight $w(x) \leq \lfloor \frac{d-1}{2} \rfloor$ are coset leaders of distinct cosets of this code.
   (b) Let C be a perfect code with $d(C) = 2t+1$. Show that the only coset leaders of C are the ones determined in part (a).
   (c) Assuming that the perfect code C in part (b) is binary, let $\widehat{C}$ be the code obtained from C by adding a parity-check digit, i.e.,

$$\widehat{C} = \left\{ (x_1, \ldots, x_n, x_{n+1}) \in \mathbb{F}_2^{n+1} : (x_1, \ldots, x_n) \in C \,, \sum_{i=1}^{n+1} x_i = 0 \right\} .$$

   Show that the weight of any coset leader of $\widehat{C}$ is less or equal than $t+1$.

3. Consider a linear code C over $\mathbb{F}_3 = \{0, 1, 2\}$ with parity-check matrix

$$H = \begin{bmatrix} 2 & 1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 & 0 \end{bmatrix} .$$

   (a) Determine the $[n, k, d]$ parameters of C.
   (b) Find a generator matrix in standard form for the code C.
   (c) What is the capacity of C for correcting erasure errors? Give a detailed justification.
   (d) Explain what to do with the following received words

$$x = 2101?? \,, \qquad y = 1???12 \qquad \text{e} \qquad z = ???210 \,.$$

4. Problem 3(c) in Exercise List 4: Find a $[7, k]$ binary linear code, with the largest possible rate, which can correct the following error vectors: 1000000, 1000001, 1100001, 1100011, 1110011, 1110111 and 1111111.