COMBINATÓRIA E TEORIA DE CÓDIGOS
Exercise List 7

26/4/2011

Problem 1.a) Exersices 12.9, 12.20 and 12.21 in R. Hill.

b) Let $C = \langle (t+1)f(t) \rangle$ be a binary cyclic code with length n, where $f(t) \mid t^n - 1$, but $f(t) \nmid t^k - 1, \forall k : 1 \leq k \leq n-1$. Show that C corrects all simple errors and also the adjacent double errors.

Problem 2. Consider binary cyclic code with length $n = 15$ generated by the polynomial $g(t) = 1 + t^3 + t^4 + t^5 + t^6$.

a) Justify that $g(t)$ is indeed the generator polynomial of this code.

b) Write a generator matrix, the check polynomial and a parity-check matrix for this code.

c) Write a generator matrix in the form $G = \begin{bmatrix} R & I \end{bmatrix}$ for this code and the corresponding parity-check matrix.

d) Use systematic coding to encode the message vector $\mathbf{m} = 010010001$.

e) Given that this code has minimum distance $d(C) = 5$, descode the received vector $\mathbf{y} = 010011000111010$, and carefully justify your procedure.

Problem 3.a) Verify that $g(t) = 2 + t^2 + 2t^3 + t^4 + t^5$ divides $t^{11} - 1$ in $\mathbb{F}_3[t]$.

b) Let C be the ternary cyclic code generated by $g(t)$. Knowing that it's a $[11, 6, 5]_3$ code (THEOREM 12.21 in R. Hill), use the Error Trapping Algorithm to descode the received vector $\mathbf{y} = 20121020112$.

c) What is the proportion of errors with weight 2 which are not corrected by this algorithm?

Problem 4. Consider the binary cyclic code $[15, 5, 7]$ with generator polynomial $g(t) = 1 + t + t^2 + t^4 + t^5 + t^8 + t^{10}$.

a) Justify that the Error Trapping Algorithm can correct all error vectors with weight ≤ 3 except for $\widehat{e} = 100001000010000$ and its cyclic shifts $\widehat{e}^j$.

b) Decode the received vector $\mathbf{y} = 111101010011101$.

c)(i) Complete this algorithm so that it also corrects the errors of the form $\widehat{e}^j, j = 0, 1, 2, 3, 4$.

[SUGESTION: Note that the syndrome of $\widehat{e}(t)$ is $1 + t^5 + \rho(t)$, where $\rho(t)$ is the remainder of the division of t^{10} by $g(t)$.]

(ii) Decode the received vector $\mathbf{y}' = 111000111100100$.

Problem 5. Consider again the binary cyclic with length $n = 15$ with generator polynomial $g(t) = 1 + t^3 + t^4 + t^5 + t^6$ in Problem 2.

a) Verift that, althougth this is a code with minimum distance 5, it corrects up to burst 3-errors. Explain carefully the meaning of that statement and justify your answer.

b) Use the Burst-Error Trapping Algorithm to decode the received vector $\mathbf{y} = 011100000111000$.