COMBINATÓRIA E TEORIA DE CÓDIGOS
Exercise List 6

26/4/2011

Exercises 12.1 – 12.17 in R. Hill

Problem 1.a) Determine the generator polynomial and the dimention of the smallest ternary cyclic code which contains the word $\mathbf{c} = 212110$.

b) What's the minimum distance of that code? Justify your answer.

Problem 2. Supose that, in $\mathbb{F}_2[t]$,

$$t^n - 1 = (t-1)g_1(t)g_2(t) \ ,$$

and that $\langle g_1(t)\rangle$ and $\langle g_2(t)\rangle$ are equivalent codes. Show that:

a) If $c(t)$ is a code word in $\langle g_1(t)\rangle$ with odd weight w, then:

(i) $w^2 \geq n$;

(ii) If, moreover, $g_2(t) = \overline{g}_1(t)$, then $w^2 - w + 1 \geq n$.

b) If n is an odd prime number, $g_2(t) = \overline{g}_1(t)$ and $c(t)$ is a code word in $\langle g_1(t)\rangle$ with even weight w, then:

(i) $w \equiv 0 \pmod 4$;

(ii) $n \neq 7 \Rightarrow w \neq 4$.

c) Show that the binary cyclic code with length 23 generated by the polynomial $g(t) = 1 + t^2 + t^4 + t^5 + t^6 + t^{10} + t^{11}$ is a perfect code $[23, 12, 7]$ — *the binary Golay Code*.