COMBINATÓRIA E TEORIA DE CÓDIGOS
Exercise List 4

5/3/2011

Exercises 7.1 - 7.9 + 7.11 (R. Hill)

Problem 1. Consider the linear code over $\mathbb{F}_{11}$ with parity-check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & X \\ 1^2 & 2^2 & 3^2 & 4^2 & 5^2 & 6^2 & 7^2 & 8^2 & 9^2 & X^2 \end{bmatrix}.$$

a) Find the parameters $[n, k, d]$ of this code. [SUGESTION : First show that in any field $\mathbb{K}$

$$\begin{vmatrix} 1 & 1 & 1 \\ a_1 & a_2 & a_3 \\ a_1^2 & a_2^2 & a_3^2 \end{vmatrix} = (a_3 - a_1)(a_2 - a_1)(a_3 - a_2), \forall a_1, a_2, a_3 \in \mathbb{K} \text{]}.$$

b) Write a generating matrix for the code;

c) **(i)** Describe a decoding algorithm for this code that can correct 1 error and detect 2 errors in any position.

(ii) Apply that algorithm to decode the received vectors

$$\mathbf{x} = 0204000910; \qquad \mathbf{y} = 0120120120.$$

Problem 2. The analogous problem to the previous one for the linear code over $\mathbb{F}_{11}$ with parity-check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & X \\ 1^2 & 2^2 & 3^2 & 4^2 & 5^2 & 6^2 & 7^2 & 8^2 & 9^2 & X^2 \\ 1^3 & 2^3 & 3^3 & 4^3 & 5^3 & 6^3 & 7^3 & 8^3 & 9^3 & X^3 \end{bmatrix};$$

Decode also the received vector

$$\mathbf{z} = 1204000910.$$

Remark: In Chapter 11 of R. Hill, read about this problem and futher generalizations.

Problem 3. **a)** Study in detail and explain the solution to Exercise 7.10 in R. Hill.

b) Find a different way to solve that exercise.

c) Find a $[7, K]$ linear code with the largest possible rate which can correct the following error vectors: 1000000, 1000001, 1100001, 1100011, 1110011, 1110111 and 1111111.