

COMBINATÓRIA E TEORIA DE CÓDIGOS

Exercise List 2

2/3/2011

Exercises 2.2 - 2.4, 2.6 - 2.11 (R. Hill)

Problem 1. a) Exercises 2.17 and 2.19 in R. Hill;

b) **(Generalization of 2.19)** The important family of Reed-Muller binary codes can be obtained as follows:

$$\forall r, m \in \mathbb{N}_0 : \begin{cases} \text{RM}(0, m) = \{\vec{0}, \vec{1}\} & \text{the binary repetition code with length } 2^m \\ \text{RM}(m, m) = (\mathbb{F}_2)^{2^m} \\ \text{RM}(r, m) = \text{RM}(r, m-1) * \text{RM}(r-1, m-1), & 0 < r < m \end{cases}$$

where $C_1 * C_2$ denotes the Plotkin Construction obtained from the codes C_1 and C_2 .

Study this family of codes by showing that the parameters of $\text{RM}(r, m)$ are: $n = 2^m$, $M = 2^{\delta(r, m)}$, where $\delta(r, m) = \sum_{i=0}^r \binom{m}{i}$, $d = 2^{m-r}$.

Problem 2. a) Exercises 2.20 - 2.22 in R. Hill;

b) **(Generalization of the Plotkin Bound)** For q -ary codes, show that

$$A_q(n, d) \leq \frac{d}{d - \theta n},$$

where $d > \theta n$ and $\theta = \frac{q-1}{q}$.