Derivadas

1.

2. (a) 9
(b) 8

3.

4.

(a) $f'(0) = 2/(\pi \mathrm{e})$ (b) $f'_d(0) = 0, \quad f'_e(0) = 1$

5. $f'_d(0) = 1$, $f'_e(0) = 1/2$.

6. (a)
(b)

7. (a) $a = 1$, $b = 2$
(b) $y = 1 + 2x$

8. Diferenciável apenas em $x = 0$ com derivada $f'(0) = 0$.

9.

(a) $-\dfrac{1}{(x-1)^2}$ (b) $\dfrac{2(x+1)^2 - 4x(x+1)}{(x+1)^4}$ (c) $-\dfrac{1}{2\sqrt{x}(1+\sqrt{x})^2} \quad (x > 0)$

(d) $\frac{3}{2}x^{1/2}\mathrm{e}^x + x^{3/2}\mathrm{e}^x$ (e) $x2^{x+1} + x^2 2^x \ln 2$ (f) $\sec^2 x - 1$

(g) $\dfrac{(1-\operatorname{sen} x)^2 + \cos x(x + \cos x)}{(1-\operatorname{sen} x)^2}$ (h) $\operatorname{sen}(2x) \quad (x \neq \frac{\pi}{2} + k\pi)$ (i) $\dfrac{1}{\operatorname{sen}^2 x(1 + \operatorname{cotan} x)^2}$

(j) $2x(1 + \ln x) + x$ (k) $\cosh^2 x + \operatorname{senh}^2 x$

10.

(a) $-\frac{2}{3}x^{-5/3} + 5^x \ln 5$ (b) $\dfrac{-2}{1+(3-2x)^2} - \dfrac{\sec^2(1/x)}{x^2}$

(c) $\dfrac{1}{\sqrt{1-(4-x)^2}} + \dfrac{\mathrm{e}^{\sqrt{x}}}{2\sqrt{x}}$ (d) $-2x\cos(x^2) - 2x(2-x^2)\operatorname{sen}(x^2) + 2\operatorname{sen}(x^3) + 6x^3\cos(x^3)$

(e) $-\dfrac{x}{\sqrt{1-x^2}}$ (f) $\dfrac{4x^3}{1+x^8} - \dfrac{4\arctan^3 x}{1+x^2}$

11.

(a) $\dfrac{x^2 + 3 - 2x(x-1)}{(x^2+3)^2}$

(b) $(x^2+1)^{1/3} + \frac{2}{3}x^2(x^2+1)^{-2/3}$

(c) $\operatorname{sen}(x^2) + 2x^2\cos(x^2)$

(d) $\dfrac{\operatorname{senh} x + x\cosh x}{x\operatorname{senh} x}$

(e) $\dfrac{1}{(1+x^2)\sqrt{1-\arctan^2 x}}$

(f) $\dfrac{\mathrm{e}^x(1+x) - \mathrm{e}^x}{(1+x)^2}$

(g) $\dfrac{-2}{\operatorname{arcsen}\left(\dfrac{x+1}{x-1}\right)\sqrt{1-\left(\dfrac{x+1}{x-1}\right)^2}(x-1)^2}$

12.

(a) $\dfrac{\ln x}{4\sqrt{x}\sqrt{\sqrt{x}+1}} + \dfrac{\sqrt{\sqrt{x}+1}}{x}$

(b) $\dfrac{2}{3}\Big(-\dfrac{2}{x}+\dfrac{1}{x^2}\Big)^{-1/3}\Big(\dfrac{2}{x^2}-\dfrac{2}{x^3}\Big)$

(c) $\dfrac{\frac{1}{3}x^{-2/3}(x^2-1)^2 - 4x^{4/3}(x^2-1)}{(x^2-1)^4}$

(d) $-\frac{3}{10}(1+\sqrt{x}\,)^{-8/5}x^{-1/2}$

(e) $\dfrac{2x(2+\sqrt{8+x^2}\,)}{\sqrt{8+x^2}}$

(f) $\dfrac{\frac{1}{3}x^{-2/3}(1+x^3)^4 - 12x^{7/3}(1+x^3)^3}{(1+x^3)^8}$

(g) $-\frac{1}{2}\big(3x+(2x)^{1/3}\big)^{-3/2}\big(3+\frac{2}{3}(2x)^{-2/3}\big)$

(h) $\dfrac{1}{3}\Big(\dfrac{1+x^3}{1-x^3}\Big)^{-2/3}\dfrac{6x^2}{(1-x^3)^2}$

13.

(a) $-\dfrac{\ln x}{2\sqrt{\arccos(x)(1-x^2)}}+\dfrac{\sqrt{\arccos x}}{x}$

(b) $\ln(3)3^x\tan(x^2)+3^x 2x\sec^2(x^2)$

(c) $-\operatorname{sen}(\mathrm{e}^x)\mathrm{e}^x\sec(x)+\cos(\mathrm{e}^x)\sec(x)\tan(x)$

(d) $\dfrac{4x^3\operatorname{sen}(x)}{\sqrt{1-x^8}}+\operatorname{arcsen}(x^4)\cos(x)$

(e) $\dfrac{\ln(2)2^{\sqrt{x}}\cosh(x)}{2\sqrt{x}}+2^{\sqrt{x}}\operatorname{senh}(x)$

(f) $\dfrac{\operatorname{senh}(x^3)}{x}+3x^2\ln(x)\cosh(x^3)$

(g) $\Big(\dfrac{2+\cos\sqrt{x}}{1+x^2}+\dfrac{\operatorname{sen}(\sqrt{x}\,)\arctan(x)}{2\sqrt{x}}\Big)\Big/\big(2+\cos\sqrt{x}\,\big)^2$

(h) $\dfrac{1-\ln(\ln x)}{x\ln^2 x}$

(i) $\dfrac{2x\mathrm{e}^{x^2}\sec x-\mathrm{e}^{x^2}\sec(x)\tan(x)}{\sec^2 x}$

(j) $\dfrac{(1-\operatorname{sen}x)\big(1-\operatorname{sen}(x^5)\big)+5x^4\cos(x^5)(x+\cos x)}{\big(1-\operatorname{sen}(x^5)\big)^2}$

(k) $\Big((\operatorname{sen}x+x\cos x)\sqrt{1+x^2}-\dfrac{x^2\operatorname{sen}x}{\sqrt{1+x^2}}\Big)\Big/(1+x^2)$

(l) $\dfrac{x^2+\tan x}{x\mathrm{e}^x+2x\sqrt{x}}+\ln x\dfrac{(2x+\sec^2 x)(\mathrm{e}^x+2\sqrt{x}\,)-(\mathrm{e}^x+x^{-1/2})(x^2+\tan x)}{(\mathrm{e}^x+2\sqrt{x}\,)^2}$

(m) $\dfrac{\operatorname{arcsen}x+x/\sqrt{1-x^2}}{x\operatorname{arcsen}x}$

(n) $\dfrac{1}{(1+x^2)\sqrt{1-\arctan^2 x}}$

(o) $\ln(4)4^x\operatorname{sen}(\operatorname{sen}x)+4^x\cos(\operatorname{sen}x)\cos x$

(p) $\exp(\arctan x\,\tan x)\Big(\dfrac{\tan x}{1+x^2}+\arctan(x)\sec^2(x)\Big)$

(q) $\sec^2(\sqrt{x}\ln x)\Big(\dfrac{\ln x}{2\sqrt{x}}+\dfrac{1}{\sqrt{x}}\Big)$

(r) $\sec^2(\mathrm{e}^x\sec x)\mathrm{e}^x\sec(x)(1+\tan x)$

(s) $\Big(\dfrac{\operatorname{sen}x}{\sqrt{1-x^2}}+\operatorname{arcsen}(x)\cos(x)\Big)\dfrac{1}{2\sqrt{\operatorname{arcsen}x\,\operatorname{sen}x}}$

(t) $\dfrac{\mathrm{e}^x\ln x-\mathrm{e}^x/x}{\ln^2 x}\dfrac{1}{2\sqrt{\mathrm{e}^x/\ln x}}$

(u) $-\dfrac{2\ln(5)5^{(x+1)/(x-1)}}{(x-1)^2}$

(v) $-\operatorname{sen}(2x)\big(\cos(\cos^2 x)\cos(\operatorname{sen}^2 x)+\operatorname{sen}(\cos^2 x)\operatorname{sen}(\operatorname{sen}^2 x)\big)$

14.

(a) $\cos\big(\ln(x^3-1)\big)\dfrac{3x^2}{x^3-1}$

(b) $\dfrac{3x}{\sqrt{2-x^2}\sqrt{x^2-1}}$

(c) $\dfrac{4x\arctan\big(\operatorname{sen}(x^2+1)\big)\cos(x^2+1)}{1+\operatorname{sen}^2(x^2+1)}$

(d) $\dfrac{3\operatorname{sen}^2(\sqrt{x}\,)\cos(\sqrt{x}\,)}{2\sqrt{x}}$

(e) $100x^9\operatorname{sen}^9(x^{10})\cos(x^{10})$

(f) $\dfrac{-\mathrm{e}^x\operatorname{sen}(\mathrm{e}^x)}{2\sqrt{\cos(\mathrm{e}^x)}}$

(g) $-\cos\big(\cos(2^x)\big)\operatorname{sen}(2^x)\ln(2)2^x$

(h) $\dfrac{4\cos(\ln^4 x)\ln^3 x}{x}$

(i) $-4x\operatorname{sen}\big(\tan^2(x^2)\big)\tan(x^2)\sec^2(x^2)$

(j) $\dfrac{\cos\big(\sqrt{\sec x}\big)\sec(x)\tan(x)}{2\sqrt{\sec x}}$

(k) $10\tan^9(\sec x)\sec^2(\sec x)\sec(x)\tan(x)$

(l) $\dfrac{\mathrm{e}^{\sqrt{\operatorname{sen} x}}\cos x}{2\sqrt{\operatorname{sen} x}}$

(m) $\frac{3}{5}\big(\ln(\cos x)\big)^{-8/5}\tan x$

(n) $16\ln^3(\sec^4 x)\tan x$

(o) $\dfrac{12\ln(\tan^6 x)\sec^2 x}{\tan x}$

(p) $\dfrac{\sec^2\sqrt{x}}{4\sqrt{x\tan\sqrt{x}}}$

(q) $10\sec^{10}(\tan x)\tan(\tan x)\sec^2 x$

(r) $\dfrac{\sec^2(\mathrm{e}^x)\mathrm{e}^x}{\tan(\mathrm{e}^x)}$

(s) $-\dfrac{\cosh(1/x^2)}{x^3\sqrt{\operatorname{senh}(1/x^2)}}$

(t) $-\dfrac{3\operatorname{senh}(1/x^3)}{2x^4\sqrt{\cosh(1/x^3)}}$

15.

(a) $\Big(\dfrac{\ln x}{2\sqrt{x}}+\dfrac{1}{\sqrt{x}}\Big)x^{\sqrt{x}}$

(b) $\Big(\dfrac{\ln(\tan x)}{1+x^2}+\dfrac{\arctan(x)\sec^2(x)}{\tan x}\Big)(\tan x)^{\arctan x}$

(c) $\Big(-\operatorname{sen}(x)\ln(\operatorname{arcsen} x)+\dfrac{\cos x}{\sqrt{1-x^2}}\Big)(\operatorname{arcsen} x)^{\cos x}$

(d) $\big(\cos(x)\ln(\operatorname{sen} x)+\cos x\big)(\operatorname{sen} x)^{\operatorname{sen} x}$

(e) $\Big(\ln(\ln x)+\dfrac{1}{\ln x}\Big)(\ln x)^x$

(f) $\Big(\dfrac{\ln(\arctan x)}{\sqrt{1-x^2}}+\dfrac{\operatorname{arcsen} x}{(1+x^2)\arctan x}\Big)(\arctan x)^{\operatorname{arcsen} x}$

(g) $\left(\Big(\ln x+\dfrac{x-1}{x}\Big)x^{x-1}\ln x+x^{x-2}\right)x^{x^{x-1}}$

16.

(a) $2|x|, \quad x\in\mathbb{R}$

(b) $0, \quad x\neq 0$

(c) 0 se $x>0$, $2\mathrm{e}^{2x}$ se $x<0$

(d) $-\mathrm{e}^{-x}$ se $x>0$, e^x se $x<0$

(e) $1/x, \quad x\neq 0$

(f) $\operatorname{sen}|x|+|x|\cos x, \quad x\in\mathbb{R}$

17. (a) 30
(b) 1/3
(c) 14
(d) 6/5

18. $-4/\pi^2$

19. $-3\operatorname{sen}(x)\cos(\cos^2 x)-3x^2\cos\big((x^3+1)^2\big)$

20.

(a) $2xf'(x^2)$ (b) $\operatorname{sen}(2x)\big(f'(\operatorname{sen}^2 x) - f'(\cos^2 x)\big)$ (c) $\dfrac{f'(x)}{1+f(x)^2} + \dfrac{f'(\arctan x)}{1+x^2}$

(d) $f'\big(f(x)\big)f'(x)$ (e) $f'(f(f(x)))f'(f(x))f'(x)$

21.

(a) $\dfrac{\operatorname{senh} x + x\cosh x}{x\operatorname{senh} x}, \quad x \neq 0$ (b) $\dfrac{x\mathrm{e}^x}{(1+x)^2}, \quad x \neq -1$

22.

23. (a) 1/12
(b) 12

24. (a) 1/2
(b) 2

25. (a) -2, $-1/2$
(b) $D_{g^{-1}} =]0, \pi[$, $D'_{g^{-1}} = \mathbb{R}$

26. $f'(1) = g'(0)\mathrm{e}^{g(0)}$, $f''(\mathrm{e}) = \big(g''(1) - g'(1) + g'(1)^2\big)\mathrm{e}^{g(1)-2}$

27. $g'(x^4\mathrm{e}^{-x})(4x^3 - x^4)\mathrm{e}^{-x}$

28. (a) Sugestão: recorde que $\cosh^2 x - \operatorname{senh}^2 x = 1$
(b)

Teoremas fundamentais

29.

(a) $\max = 5, \quad \min = 1$ (b) $\max = 2, \quad \min = 0$ (c) $\max = \sqrt{2}, \quad \min = 0$

30. $200\,\mathrm{m} \times 400\,\mathrm{m}$

31.

32. (a)
(b) $\sqrt{2}/2$

33.

34.

35. Sugestão: intervalo $[0, 1]$

36. Sugestão: intervalo $[0, \pi/2]$

37. (a) Sugestão: intervalos $[-1, 0]$, $[0, 1]$ e $[1, 10]$
(b) Sugestão: intervalos $[0, 1]$ e $[1, 2]$

38. (a) Sugestão: para $x > 1$ temos $c \in]1, x[\Rightarrow 1/x < 1/c < 1$
(b) Sugestão: $\mathrm{e}^c > 1$ para $c > 0$ e $\mathrm{e}^c < 1$ para $c < 0$
(c) Sugestão: intervalo $[0, x]$
(d) Sugestão: intervalo $[1, x]$

39. (a) Verdadeiro
(b) Falso
(c) Verdadeiro

40. (a) Sugestão: $f(x) = \operatorname{sen} x$ e note que $0 < \cos c < 1$
(b) Sugestão: $f(x) = \cos x$ e use a alínea (a)
(c) Sugestão: $f(x) = \operatorname{sen} x$ e use a alínea (b)
Erro inferior a 0,001.

41. Sugestão: aplique o Teorema de Rolle a $f(x) - x$ e a $\big(f(x) - x\big)'$.

42. Sugestão: aplique primeiro o Teorema de Rolle à função $f(x) - x^2$ e depois o Teorema de Bolzano à função $f'(x)$.

43.* Aplique o Teorema de Lagrange a f para mostrar que existem pontos $x_1, x_2 \in \mathbb{R}$ tais que $f'(x_1) < \alpha < f'(x_2)$.

44. Sugestão: $|\cos c| \leqslant 1$.

45. Sugestão: se $x < c < y$ então $\mathrm{e}^x < \mathrm{e}^c < \mathrm{e}^y$.

46. Sugestão: $c = \max\limits_{c \in [a,b]} |f'(c)|$.