

7. Funções

1.

- (a) ímpar
- (b) estritamente crescente, minorada, $\inf = 0$
- (c) limitada, par, $\sup = \max = 1$, $\inf = 0$
- (d) limitada, estritamente decrescente, $\inf = 0$, $\sup = 1$
- (e) $a \neq 1$: minorada, $\inf = 0$, estr. crescente se $a > 1$, estr. decrescente se $a < 1$;
 $a = 1$: constante, $\inf = \min = \sup = \max = 1$
- (f) estritamente crescente, ímpar
- (g) estr. crescente, majorada, $\sup = 0$
- (h) estr. crescente
- (i) estr. crescente, limitada, $\inf = \min = 0$, $\sup = \max = \pi/2$

2. (a)

(b) $] -\infty, -1[\cup [-\frac{1}{2}, \frac{1}{2}] \cup] e, +\infty[$

(c) $f^{-1}(y) = -\sqrt{-y}$ se $y < -1$, $f^{-1}(y) = 2y$ se $-\frac{1}{2} \leq y \leq \frac{1}{2}$, $f^{-1}(y) = \ln y$ se $y > e$

3. (a) $[0, 1]$: crescente, $\sup = \max = \pi$, $\inf = \min = \pi/2$; $]1, \sqrt{3}]$: decrescente, $\sup = \pi/4$, $\inf = \min = \pi/6$; $] \sqrt{3}, +\infty[$: crescente, $\sup = +\infty$, $\inf = 3$.

(b) $\sup = +\infty$, $\inf = \min = -1$, $D'_f = [-1, 0[\cup [\pi/6, \pi/4[\cup [\pi/2, +\infty[$

(c) Não: $f(1) = f(\sqrt{\pi})$.

(d) $f^{-1}(y) = \arcsen(-\sqrt{-y})$ se $-1 \leq y < 0$, $f^{-1}(y) = \cotan y$ se $\pi/6 \leq y < \pi/4$, $f^{-1}(y) = \cos^2 y$ se $\pi/2 \leq y \leq \pi$.

4.

(a) $x = \ln 2$

(b) $x = 0$

5.

(a) $g \circ p$

(b) $f \circ q$

(c) $f \circ g$

(d) $q \circ p$

(e) $f \circ f$

(f) $q \circ g$

(g) $f \circ q \circ f$

(h) $f \circ p$

(i) $p \circ q \circ g$

6.

(a) $]0, +\infty[$

(b) $]1, +\infty[$

(c) $] -\infty, 1[$

(d) $\mathbb{R} \setminus \{k\pi/2 : k \in \mathbb{Z}\}$

(e) $[0, 1[$

(f) $[-1, +\infty[$

(g) $[-2, 2]$

(h) $\mathbb{R} \setminus \{k\pi/2 : k \in \mathbb{Z}\}$

(i) $[-1, 0] \cup \{1\}$

(j) $\{\pi/2 + 2k\pi : k \in \mathbb{Z}\}$

(k) $] -\infty, 0]$

(l) $] -\infty, -1] \cup [1, +\infty[$

(m) $[1 - \sqrt{2}, 1 + \sqrt{2}]$

(n) $]1, +\infty[$

(o) $] -\infty, -1] \cup [1, +\infty[$

(p) $\mathbb{R} \setminus \{-1, 1\}$

(q) $[-1, \text{sen } 1[$

(r) $] -2, 2[$

(s) $] \cos 1, 1]$

(t) $[1, e^2]$

7. Isolados: (i) $\{1\}$; (j) $\{\pi/2 + 2k\pi : k \in \mathbb{Z}\}$; outras alíneas: \emptyset . Pontos de acumulação: (a) $[0, +\infty[$ (b) $[1, +\infty[$ (c) $] -\infty, 1]$ (d) \mathbb{R} (e) $[0, 1]$ (f) D_f (g) D_f (h) \mathbb{R} (i) $[-1, 0]$ (j) \emptyset (k) D_f (l) D_f (m) D_f (n) $[1, +\infty[$ (o) D_f (p) \mathbb{R} (q) $[-1, \text{sen } 1]$ (r) $[-2, 2]$ (s) $[\cos 1, 1]$ (t) D_f .

8.

(a) $f^{-1}(y) = \sqrt{\ln y + 2}$, $D_{f^{-1}} =]e^{-2}, +\infty[$ (b) $f^{-1}(y) = -\sqrt{\ln y + 2}$, $D_{f^{-1}} = [e^{-2}, +\infty[$
 (c) $f^{-1}(y) = \frac{1}{2} \arccos y$, $D_{f^{-1}} =]-1, 1[$ (d) $f^{-1}(y) = \arctan y + 1$, $D_{f^{-1}} = \mathbb{R}$
 (e) $f^{-1}(y) = \arcsen(y/2)$, $D_{f^{-1}} =]-2, 2[$ (f) $f^{-1}(y) = \pi - \arcsen(y/2)$, $D_{f^{-1}} = [-2, 2]$

9.

(a) $\sqrt{3}/2$	(b) $-\sqrt{2}/2$	(c) $-\sqrt{3}/3$
(d) $\pi/2$	(e) 0	(f) $2\pi/3$
(g) $-\pi/6$	(h) $\pi/3$	(i) $5\pi/6$
(j) $\pi/4$	(k) $\pi/4$	(l) $-\pi/3$
(m) $\pi/6$	(n) $\pi/3$	(o) $-\pi/6$
(p) $-\pi/4$	(q) $\pi/3$	(r) $-\pi/3$
(s) $\pi/3$	(t) $\pi/6$	(u) $\pi/6$
(v) $-\pi/6$	(w) $\pi/4$	(x) $\pi/4$
(y) $\pi/3$	(z) $-\pi/4$	

10.

(a) (b)
 (c) Sugestão: escreva $\theta = \arcsen(x)$, justifique que $\cos \theta \geq 0$ e recorde que $\cos^2 \theta + \text{sen}^2 \theta = 1$. (d)
 (e) Sugestão: use (b) e (c). (f)

11.

(a) $-\frac{1}{2}$	(b) $\pm\sqrt{2}$
(c) $\pm\frac{1}{2}$	(d) $x \geq 2$
(e) $x > 2\sqrt{3}$	(f) $x < 0$
(g) $x = 0$	(h) $0 < x \leq 1$
(i) $x \leq -2 \vee x \geq 2$	(j) $1 < x \leq e$
(k) $x \leq 0$	(l) $[-2, -\sqrt{3}] \cup [\sqrt{3}, 2]$

12.

(a) $x^3 - \sqrt{x}$ se $x \geq 1$, $\sqrt{x} - x^3$ se $0 \leq x < 1$	(b)	(c)	(d)
(e)	(f)	(g)	(h)

13.

(a) $1 - \ln 2$ porque $\ln x \leq x - 1$ (b) $2 - \arcsen 0,8$ (c) $2 - \arccos 0,6$ porque $\arccos x < \pi/2 < 2$ para $x > 0$
 (d) $2 - \arctan 10$ (e) $\tan 0,2 - 0,2$ (f) $1 - \arctan 0,8$

14.

15.

16. (a)
 (b)

17.

18. A primeira e a última

19. (a) (i)
(ii)
(iii)
(iv)
(v)
(b)
(c) É a soma de funções crescentes
(d) Sugestão: note que $e^{2y} = (e^y)^2$ e resolva a equação de segundo grau.
20. Sugestão: pode definir f por ramos.
21. Não
22. (a)
(b)
(c)

8. Limites de funções

1.

2.

(a) $+\infty$

(b) $-\infty$

(c) 0

(d) 1

(e) $\frac{1}{2}$

(f) -3

(g) 0

(h) 0

(i) 1

3.

(a) $f(-3^+) = -\infty, f(-3^-) = +\infty$

(b) $f(2^+) = \frac{1}{5}, f(2^-) = -\frac{1}{5}$

4.

(a) 1

(b) 1

(c) -1

5. $f(0^+) = 1, f(0^-) = -1$

6. Diverge

7.

(a) $\delta = \varepsilon/5$

(b) $\delta = 1/\ln \varepsilon \quad (\varepsilon < 1)$

(c) $\delta = -1/\ln \varepsilon \quad (\varepsilon < 1)$

(d) $\delta = (\pi/2) - \arctan(1/\varepsilon)$

(e) $\delta = \sqrt{\varepsilon}$

(f) $\delta = \varepsilon$

(g) $\delta = \varepsilon^2$

(h) $\delta = -1/\ln \varepsilon \quad (\varepsilon < 1)$

(i) $\delta = \varepsilon^3$

(j) $\delta = \varepsilon$

8. $\lim_{x \rightarrow +\infty} f(x)$ não existe, $\lim_{x \rightarrow -\infty} f(x) = 0$ e $\lim_{x \rightarrow 1/2} f(x) = 2$.

9.* Sugestão: mostre que $\forall m \exists x f(x) > m$ e portanto f não é majorada.

10. (a)* Sugestão: veja o exercício anterior

(b) 2 exemplos: $f(x) = 1/x$ se $x \in \mathbb{Q}$, $f(x) = -1/x$ se $x \notin \mathbb{Q}$; $g(x) = \text{sen}(1/x)/x$

11.* Sugestão: mostre que se $b \in D_f$ então existe um $x < a$ tal que $f(x) > f(b)$ e conclua que $b < a$.

12.* Sugestão: dado um $y < 0$ mostre que existe um $x \in]y, 0[$ tal que $f(x) < f(y)$.

13.* Se T é o período de f , para cada $x \in D_f$ considere a restrição de f a $\{x + nT : n \in \mathbb{Z}\}$