

Métodos de primitivação

Primitivação por partes

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| (a) $x e^x + \frac{1}{2} e^{2x} - e^x$ | (b) $\operatorname{sen} x - x \cos x$ |
| (c) $\frac{1}{2} e^x (\operatorname{sen} x - \cos x)$ | (d) $-e^{-x}(x^3 + 3x^2 + 6x + 6)$ |
| (e) $(x^2 + 2) \cosh x - 2x \operatorname{senh} x$ | (f) $x \arctan x - \frac{1}{2} \ln(x^2 + 1)$ |
| (g) $x \arcsen x + \sqrt{1 - x^2}$ | (h) $\frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2}$ |
| (i) $\frac{1}{2}(x^2 + 1) \arctan x - \frac{1}{2} x$ | (j) $\frac{1}{4}(x^2 + 1)^2 \arctan x - \frac{1}{12} x^3 - \frac{1}{4} x$ |
| (k) $x \ln \left(\frac{1}{x} + 1 \right) + \ln(x + 1)$ | (l) $x^3 \left(\frac{1}{3} \ln^2 x - \frac{2}{9} \ln x + \frac{2}{27} \right)$ |
| (m) $x(\ln^2 x - 2 \ln x + 2)$ | (n) $x(\ln^3 x - 3 \ln^2 x + 6 \ln x - 6)$ |
| (o) $\frac{1}{2} \operatorname{sen}(2x) \ln(\tan x) - x$ | (p) $-\cos(x) \ln(1 + \operatorname{sen} x) + \cos(x) + x$ |
| (q) $-(1 - x^2)^{3/2} \arcsen(x) - \frac{1}{3} x^3 + x$ | (r) $\frac{x \ln x}{x + 1} - \ln(x + 1)$ |
| (s) $\frac{1}{2} (\cos(x) \operatorname{senh}(x) + \operatorname{sen}(x) \cosh(x))$ | (t) $\frac{3^x (\operatorname{sen} x + \ln 3 \cos x)}{1 + \ln^2 3}$ |
| (u) $\frac{2}{3} x^{3/2} \arctan(1/\sqrt{x}) - \frac{1}{3} \ln(x + 1) + \frac{1}{3} x$ | (v) $\frac{2}{3} x^{3/2} \arctan(\sqrt{x}) + \frac{1}{3} \ln(x + 1) - \frac{1}{3} x$ |
| (w) $\frac{1}{2} x (\operatorname{sen}(\ln x) + \cos(\ln x))$ | (x) $\frac{x^{n+1}}{n + 1} \ln x - \frac{x^{n+1}}{(n + 1)^2}$ |

2.

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|-------------------------------------|--|
| (a) $\frac{1}{2} e^{x^2} (x^2 - 1)$ | (b) $-\frac{1}{x} \operatorname{sen} \frac{1}{x} - \cos \frac{1}{x}$ |
|-------------------------------------|--|

3.

4. (a)

(b) Sugestão: $\frac{1}{(1 + x^2)^k} = \frac{1}{(1 + x^2)^{k-1}} - \frac{x^2}{(1 + x^2)^k}$.

(c)

5.

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|-------------------------------|----------|--|-------------------------|
| (a) $\frac{1}{2} (\ln 2 - 1)$ | (b) -1 | (c) $(\pi \operatorname{senh}(1/2) - 1)/(1 + \pi^2)$ | (d) $\frac{1}{4} \ln 2$ |
|-------------------------------|----------|--|-------------------------|

6. Sugestão: use integração por partes.

Primitivação de funções racionais

7.

- (a) $\ln|1-x|$ (b) $-\frac{1}{2(x-3)^2}$ (c) $\frac{1}{2}\ln(x^2+1) + \arctan x$
 (d) $\frac{1}{2}\ln(1+(x-1)^2) + \arctan(x-1)$ (e) $\ln(x^2+4) + \frac{1}{2}\arctan(x/2)$ (f) $\arctan(x+1)$
 (g) $-\frac{1}{x+2} + \frac{1}{2(x+2)^2}$ (h) $\frac{1}{2}\ln(a^2+x^2) + \frac{1}{a}\arctan(x/2)$ (i) $\frac{2}{\sqrt{3}}\arctan((2x+1)/\sqrt{3})$

8.

- (a) $\ln|x| - \ln|x+1|$ (b) $-\frac{2}{x-1} - \ln|x-1| + \ln|x|$
 (c) $-\ln|x| + \ln(x^2+4) + \frac{1}{2}\arctan(x/2)$ (d) $1/x - \ln|x| + 2\ln|x-1|$
 (e) $\frac{1}{4}x^4 + \frac{1}{2}x^2 + \frac{1}{2}\ln|x^2-1|$ (f) $-2/(x+2) + \ln|x+2| - \ln|x+1|$
 (g) $\frac{1}{2}x^2 + 1/(x+1) + \ln|x+1|$ (h) $x + \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| - \frac{1}{2}\arctan x$
 (i) $\frac{1}{2}(\ln|x-2| - \ln|x+2| + \ln(x^2+4)) + \arctan(\frac{x}{2})$ (j) $\ln|x| + \ln(x^2+2x+2) - 4\arctan(x+1)$
 (k) $-\frac{1}{2}(x-1)^{-2} - 2(x-1)^{-1}$ (l) $-1/(x+1) + \ln|x+1| + \ln|x-3|$
 (m) $-1/(x+1) + \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1|$ (n) $-\frac{1}{5}\ln|x-2| + \frac{1}{10}\ln(x^2+1) + \frac{2}{5}\arctan x$
 (o) $\ln|x| - 2/(x-1)$ (p) $\ln|x-3| - 1/(x+1)$
 (q) $\frac{1}{2}\ln(x^2+2x+2) + \ln|x+1|$ (r) $\ln|x+1| + \ln(x^2+x+1) - \frac{2}{\sqrt{3}}\arctan(\frac{2x+1}{\sqrt{3}})$

9. $\frac{3}{x} + \ln \frac{(x-1)^2}{x(x+1)} - 3$

Mudança de variável

10.

- (a) $\pi/12$ (b) $\ln \sqrt{\frac{(2-\sqrt{3})(2+\sqrt{2})}{(2+\sqrt{3})(2-\sqrt{2})}}$
 (c) $1 - \frac{\sqrt{3}}{2} + \frac{\pi}{6}$ (d) $\frac{1}{2}\ln \frac{3}{2}$
 (e) $\frac{\pi}{8} + \frac{1}{4}\ln 2$ (f) $4\ln(15/7)$

11.

- (a) $-2\ln|\sqrt{x}+2| + \ln|x+1| + \arctan \sqrt{x}$ (b) $-2\arctan \sqrt{1-x}$
 (c) $\ln|1-\sqrt{2x+1}| - \ln|1+\sqrt{2x+1}|$ (d) $2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\ln(\sqrt[6]{x}+1)$
 (e) $\ln|1-\sqrt[4]{x+1}| - \ln|1+\sqrt[4]{x+1}| + 2\arctan \sqrt[4]{x+1}$ (f) $2\operatorname{arcsen}(e^{x/2})$
 (g) $-1/(2e^x+2) - \frac{1}{4}\ln|e^x+1| + \frac{1}{4}\ln|e^x-1|$ (h) $\ln|\sqrt{1+e^x}-1| - \ln|\sqrt{1+e^x}+1|$
 (i) $\frac{1}{4}\ln|\ln x+2| - \frac{1}{4}\ln|\ln x-2|$ (j) $\ln|\ln x| - \ln|\ln x-1|$
 (k) $-\frac{1}{\operatorname{sen} x} - \frac{1}{2}\ln|1-\operatorname{sen} x| + \frac{1}{2}\ln|1+\operatorname{sen} x|$ (l) $\ln|\sec x + \tan x|$
 (m) $\frac{1}{2\cos^2 x} - \frac{1}{4}\ln|1-\operatorname{sen} x| + \frac{1}{4}\ln|1+\operatorname{sen} x|$ (n) $\ln|\operatorname{sen} x| - \ln|\operatorname{sen} x+1|$
 (o) $\ln|2-\cos x| - \ln|1-\cos x|$ (p) $-2\arctan(\cos x)$
 (q) $\frac{1}{2-2\operatorname{sen} x} - \frac{1}{4}\ln|1-\operatorname{sen} x| + \frac{1}{4}\ln|1+\operatorname{sen} x|$ (r) $\frac{1}{2\cos x+2} + \frac{1}{4}\ln|1-\cos x| - \frac{1}{4}\ln|1+\cos x|$
 (s) $\arctan(\operatorname{senh} x)$ (t) $\frac{2}{5}x - \frac{1}{10}\ln(\sec^2 x) + \frac{1}{5}\ln|\tan x+2|$
 (u) $\frac{1}{2}(x\sqrt{1-x^2} + \operatorname{arcsen} x)$ (v) $-(1-x^2)^{3/2}/(3x^3)$
 (w) $\sqrt{x^2-1}/x$ (x) $2\operatorname{arcsen} \sqrt{x}$
 (y) $\ln(x + \sqrt{x^2+1})$ (z) $\frac{1}{2}(x-2)\sqrt{x^2-1} + \frac{1}{2}\ln(\sqrt{x^2-1}+x)$

12.

- (a) $\int \sec^3 t \, dt$ (b) $\frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2} \operatorname{argsenh} x$
 (c) $\int \sec(t) \tan^2(t) \, dt \quad (x \geq 1)$ (d) $\frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2} \operatorname{argcosh} x \quad (x \geq 1)$
 (e) $\int \frac{1}{\operatorname{sen} t} \, dt$ (f) $\int \frac{1}{t^2-1} \, dt$
 (g) $\int \frac{1}{\operatorname{sen} t} \, dt$ (h) $\int \frac{1}{t^2-1} \, dt$ (i) $\int \frac{1}{\operatorname{senh} t} \, dt$
 (j) $\int \frac{1}{t^2+1} \, dt$ (k) $\int dt \quad (x \geq 1)$ (l) $\int \frac{1}{\operatorname{cosh} t} \, dt \quad (x \geq 1)$

13. (a)
(b)

14. Sugestão: comece por integrar por partes.

Exercícios suplementares

15. Sugestões:

- (a) por partes seguido de $t = x^2$ (b) Fórmulas trigonométricas (c) Por partes
 (d) $t = \ln x$ (e) Por partes (f) Fracções simples
 (g) $t = \ln x$ (h) $t = 1/x$ (i) $t = \operatorname{sen} x$
 (j) $t = \ln x$ (k) $u = \frac{1}{2}(x^2 + 1), \quad v = \operatorname{arctan}^2 x$ (l) Por partes
 (m) $t = \cos x$ (n) $t = \cos x$ (o) Por partes duas vezes

16. Sugestões:

- (a) (b) $t = e^x$ (c) Separe numa soma
 (d) Fracções simples (e) $t = \sqrt{x}$ (f) Por partes
 (g) $t = \cos x$ (h) Por partes (i) $t = \operatorname{arctan} x$
 (j) Por partes (k) Por partes (l) Por partes
 (m) Por partes (n) $t = e^x$ (o) Fracções simples