Séries de potências

Domínio de convergência

1. (a), (c)

2.

(a) $AC:\ ]-2,2[\,;\quad SC:\ \varnothing$ (b) $AC:\ ]-4,0[\,;\quad SC:\ -4$ (c) $AC:\ \left]-\frac{1}{2},\frac{1}{2}\right[\,;\quad SC:\ -\frac{1}{2}$

(d) $AC:\ \left]-\frac{1}{4},\frac{1}{4}\right[\,;\quad SC:\ \frac{1}{4}$ (e) $AC:\ ]-1,1[\,;\quad SC:\ \varnothing$ (f) $AC:\ ]0,2[\,;\quad SC:\ \varnothing$

(g) $AC:\ ]-1,1[\,;\quad SC:\ -1$ (h) $AC:\ ]2,4[\,;\quad SC:\ 2$ (i) $AC:\ ]-3,5[\,;\quad SC:\ \varnothing$

(j) $AC:\ ]-2,4[\,;\quad SC:\ \varnothing$ (k) $AC:\ ]a-|a|,a+|a|[\,;\quad SC:\ \varnothing$ (l) $AC:\ ]-2,0[\,;\quad SC:\ -2$

(m) $AC:\ [0,2]$ (n) $AC:\ ]0,2[\,;\quad SC:\ 2$ (o) $AC:\ [-4,-2]$

(p) $AC:\ ]0,4[\,;\quad SC:\ \varnothing$ (q) $AC:\ \mathbb{R}$

3.

(a) $AC:\ ]-4,0[\,;\quad SC:\ \varnothing$ (b) $AC:\ \left]0,\frac{4}{3}\right[\,;\quad SC:\ 0$ (c) $AC:\ \left]\frac{1}{5},\frac{3}{5}\right[\,;\quad SC:\ \frac{1}{5}$

(d) $AC:\ [1,2]$ (e) $AC:\ ]0,1[\,;\quad SC:\ 1$ (f) $AC:\ \left]\frac{1}{3},1\right[\,;\quad SC:\ \varnothing$

4.

(a) $AC:\ \left]-\frac{1}{2},+\infty\right[\,;\quad SC:\ \varnothing$ (b) $AC:\ ]1,+\infty[\,;\quad SC:\ 1$ (c) $AC:\ \mathbb{R}$

(d) $AC:\ ]-1,1[\,;\quad SC:\ -1,1$ (e) $AC:\ [-\frac{2}{5},0]$ (f) $AC:\ \mathbb{R}$

(g) $AC:\ \{0\};\quad D:\ \mathbb{R}\backslash\{0\}$

5.

(a) $AC:\ ]-4,4[\,;\quad SC:\ \varnothing$ (b) $AC:\ ]-2,2[\,;\quad SC:\ -2$ (c) $AC:\ ]-5,-1[\,;\quad SC:\ -5$

(d) $AC:\ ]-2,6[\,;\quad SC:\ \varnothing$ (e) $AC:\ ]-2,0[\,;\quad SC:\ -2$ (f) $AC:\ [0,1]$

(g) $AC:\ \left]-\frac{1}{3},1\right[\,;\quad SC:\ \varnothing$ (h) $AC:\ \mathbb{R}$ (i) $AC:\ ]-\mathrm{e},\mathrm{e}[\,;\quad SC:\ \varnothing$

(j) $AC:\ ]-4,4[\,;\quad SC:\ \varnothing$

6. Sugestão: termos pares + termos ímpares. AC: $\left]-\frac{1}{4},\frac{1}{4}\right[$; SC: $\varnothing$

7. (a) simples
(b) AC: $]-3,3[$. D: $]-\infty,-3[\,\cup\,[3,+\infty[$
.

8. (a) AC (b) AC (c) D (d) C

9. Critério da razão

10. (a) 1

Integração e derivação. Séries de Taylor

11. $f'(x)=\displaystyle\sum_{k=1}^{+\infty}\frac{\sqrt{k}\,x^{k-1}}{(k-1)!}$ e $\displaystyle\int f(x)\,\mathrm{d}x = C+\sum_{k=0}^{+\infty}\frac{\sqrt{k}\,x^{k+1}}{(k+1)!}$

12. $f^{(48)}(-1/3) = \dfrac{3^{48}48!}{24!\sqrt{24}}$

13.

(a) $\sum_{k=0}^{\infty} 3x^{4k} = 3 + 3x^4 + \cdots, \quad |x| < 1$

$f^{(4k)}(0) = 3 \cdot (4k)!$ e $f^{(n)}(0) = 0$ se n não é múltiplo de 4. Mínimo

(b) $\sum_{k=0}^{\infty} (-9)^k x^{2k} = 1 - 9x^2 + \cdots, \quad |x| < 1/3, \quad f^{(2k)}(0) = (-9)^k (2k)!, \quad f^{(2k+1)}(0) = 0,$ Máximo

(c) $\sum_{k=0}^{\infty} (-4)^k x^{k+1} = x - 4x^2 + \cdots, \quad |x| < 1/4, \quad f^{(k)}(0) = (-4)^{k-1} k!$

(d) $\sum_{k=0}^{\infty} \dfrac{1}{(-9)^{k+1}} x^{2k+1} = \dfrac{x}{9} - \dfrac{x^3}{9^2} + \cdots, \quad |x| < 3, \quad f^{(2k)}(0) = 0, \quad f^{(2k+1)}(0) = \dfrac{(2k+1)!}{(-9)^{k+1}}$

(e) $\sum_{k=2}^{\infty} \dfrac{(-1)^k k(k-1)}{2} x^k = x^2 - 3x^3 + \cdots, \quad |x| < 1$

$f'(0) = 0, \quad f^k(0) = \dfrac{(-1)^k k(k-1)k!}{2} \ (k > 1),$ Mínimo

(f) $\ln 5 + \sum_{k=0}^{\infty} \dfrac{-1}{5^{k+1}(k+1)} x^{k+1} = \ln 5 - \dfrac{x}{5} - \dfrac{x^2}{50} - \cdots, \quad f^{(k)}(0) = -\dfrac{(k-1)!}{5^k}$

(g) $\sum_{k=1}^{\infty} \dfrac{k}{2^{k+1}} x^{k+2} = \dfrac{x^3}{4} + \dfrac{x^4}{4} + \cdots, \quad |x| < 2, \quad f'(0) = f''(0) = 0, \quad f^{(k)}(0) = \dfrac{k!(k-2)}{2^{k-1}},$ não é extremo

(h) $\sum_{k=0}^{\infty} \dfrac{(-1)^k}{3^{2k+1}(2k+1)} x^{2k+1} = \dfrac{x}{3} - \dfrac{x^3}{27} + \cdots, \quad |x| < 3, \quad f^{(2k)}(0) = 0, \quad f^{(2k+1)}(0) = \dfrac{(-1)^k (2k)!}{3^{2k+1}}$

(i) $\ln 3 + \sum_{k=0}^{\infty} \dfrac{(-1)^k}{3^{k+1}(k+1)} x^{k+1} = \ln 3 + \dfrac{x}{3} - \dfrac{x^2}{18} + \cdots, \quad |x| < 3, \quad f^{[k)} = \dfrac{(-1)^{k-1}(k-1)!}{3^k}$

(j) $\sum_{k=0}^{\infty} \dfrac{2}{2k+1} x^{2k+1}, \quad |x| < 1$

(k) $\sum_{k=0}^{\infty} \dfrac{(-1)^k}{k!} x^{k+2}, \quad x \in \mathbb{R},$ mínimo

(l) $\sum_{k=0}^{\infty} \dfrac{(-1)^k}{2k+1} x^{2k+2}, \quad |x| < 1,$ mínimo

(m) $\sum_{k=0}^{\infty} \dfrac{(-1)^k}{(2k+1)!} x^{8k+4}, \quad x \in \mathbb{R},$ mínimo

(n) $\sum_{k=0}^{\infty} \dfrac{(-1)^k 2^{2k}}{(2k)!} x^{2k+1}, \quad x \in \mathbb{R}$

(o) $\sum_{k=0}^{\infty} \left(\dfrac{1}{k!} + \dfrac{1}{(2k)!} \right) (-1)^k x^{2k}, \quad x \in \mathbb{R},$ máximo

(p) $\sum_{k=0}^{\infty} \dfrac{(\ln 2)^k}{k!} x^k, \quad x \in \mathbb{R}$

(q) $\sum_{n=1}^{\infty} (-1)^{n-1} n x^{2n-1}, \quad |x| < 1$

(r) $\sum_{n=1}^{\infty} nx^{n-1}, \quad |x| < 1$

(s) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1}}{(2n+1)!} x^{2n+3}, \quad x \in \mathbb{R},$ não é extremo

(t) $\sum_{n=0}^{\infty} 2^n x^{n+4}, \quad |x| < \frac{1}{2},$ mínimo

(u) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{3n+4}, \quad |x| < 1,$ mínimo

14.

(a) $\sum_{n=0}^{\infty} \frac{\mathrm{e}\, 2^n}{n!} x^n, \quad x \in \mathbb{R}$

(b) $\sum_{n=0}^{\infty} (-2)^n x^{n+1}, \quad |x| < \frac{1}{2}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x+1)^{4n}, \quad x \in \mathbb{R},$ máximo

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!(2n+1)} x^{2n+1}, \quad x \in \mathbb{R}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!(4n+3)} x^{4n+3}, \quad x \in \mathbb{R},$ não é extremo

(f) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)(2n+3)} x^{4n+6}, \quad |x| < 1,$ mínimo

(g) $\sum_{n=0}^{\infty} (-1)^{n-1} n x^{n-1}, \quad |x| < 1$

(h) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} (x-1)^n, \quad |x-1| < 2$

(i) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{4n+2}, \quad |x| < 1,$ mínimo

(j) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{2n+2}, \quad |x| < 1,$ mínimo

(k) $\sum_{n=0}^{\infty} \frac{\mathrm{e}}{n!} (x-1)^{n+1}, \quad x \in \mathbb{R}$

(l) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}(n+1)} (x-2)^{n+1}, \quad |x-2| < 2$

15. $f(x) = \frac{16x^4}{9-12x^2}, \quad |x| < \sqrt{3}/2$

16.

(a) $\frac{x^2}{1+x}, \quad |x| < 1$ (b) $\frac{2x-2}{3-x}, \quad |x-1| < 2$ (c) $\frac{x^2}{9-3x^2}, \quad |x| < \sqrt{3}$

(d) $\frac{x}{8-4x}, \quad |x| < 2$ (e) $\mathrm{e}^{2x}, \quad x \in \mathbb{R}$ (f) $\mathrm{e}^{x/3}, \quad x \in \mathbb{R}$

(g) $\frac{1}{2}\mathrm{e}^{-x} - 1, \quad x \in \mathbb{R}$ (h) $\mathrm{sen}(x+1), \quad x \in \mathbb{R}$ (i) $\cos(2x^2), \quad x \in \mathbb{R}$

(j) $\mathrm{e}^{x^2}, \quad x \in \mathbb{R}$ (k) $x\mathrm{e}^{x/2}, \quad x \in \mathbb{R}$

17.

(a) Sugestão: comece por pôr um x em evidência e primitive (b) Sugestão: comece por derivar

18.

(a) (b) (c)

19. (a) $R = 3$, $]-2, 4[$

(b) $g(1) = 0$, $g''(1) = 1/(18\sqrt{2})$, $x + g'(x) = 1 + (x-1) + \sum_{n=1}^{\infty} \frac{\sqrt{n}}{3^n} x^{n-1}$

20.* Sugestão: escreva a função como uma série de potências na origem, primitive a série e note que após substituir nos extremos a série resultante é alternada.

(a) (b) (c) (d)

21.* (a) Sugestão: considere x constante, escreva a série de Taylor de e^{xt^2} na origem e primitive.

(b) Sugestão: escreva a série de Taylor de $t^2 e^{xt^2}$ na origem e primitive.

22.

(a) $c_k = 1/k!$ (b) $c_{2k} = 0$, $c_{2k-1} = (-1)^k/(2k-1)!$

23.*

24.*

25.*