1^a Ficha de Avaliação de CDI-I; LMAC, MEBiom e MEFT

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Resolução

1) $|2x+1| < |4x-7| \iff |2x+1|^2 < |4x-7|^2 \iff$
$\iff (2x+1)^2 < (4x-7)^2 \iff 4x^2+4x+1 < 16x^2-56x+49 \iff$
$\iff 0 < 12x^2-60x+48 \iff 0 < x^2-5x+4$.

Ora,
$x^2-5x+4=0 \iff x=1 \vee x=4$.
Sendo assim,
$0 < x^2-5x+4 \iff x \in \left]-\infty,1\right[\cup \left]4,+\infty\right[$.
Logo,

$$A = \left]-\infty,1\right[\cup \left]4,+\infty\right[\ .$$

2) Pretende-se mostrar, por Indução Matemática, que:

$$\forall n \in \mathbb{N} : \sum_{k=1}^{n} \frac{5-2k}{3^k} = 1 + \frac{n-1}{3^n} \ .$$

$$n = 1 :$$

$\sum_{k=1}^{n} \frac{5-2k}{3^k} = \sum_{k=1}^{1} \frac{5-2k}{3^k} = \frac{5-2}{3} = 1,$
$1 + \frac{n-1}{3^n} = 1 + \frac{0}{3} = 1$.
Logo, $P(n)$ é satisfeita para $n=1$.

Hipótese de Indução (H.I.): $\underbrace{\sum_{k=1}^{n} \frac{5-2k}{3^k} = 1 + \frac{n-1}{3^n}}_{P(n)}$.

Tese de Indução: $\underbrace{\sum_{k=1}^{n+1} \frac{5-2k}{3^k} = 1 + \frac{n}{3^{n+1}}}_{P(n+1)}$.

Ora,
$\sum_{k=1}^{n+1} \frac{5-2k}{3^k} = \sum_{k=1}^{n} \frac{5-2k}{3^k} + \frac{5-2(n+1)}{3^{n+1}} \underset{\textbf{H.I.}}{=} 1 + \frac{n-1}{3^n} + \frac{5-2(n+1)}{3^{n+1}} =$
$= 1 + \frac{3(n-1)+5-2(n+1)}{3^{n+1}} = 1 + \frac{3n-3+5-2n+-2}{3^{n+1}} = 1 + \frac{n}{3^{n+1}}$.

3) $\lim\limits_{x\longrightarrow 0^-} \sinh\left(\frac{1}{x^2}\right) = \lim\limits_{x\longrightarrow 0^-} \frac{e^{\frac{1}{x^2}} - e^{-\frac{1}{x^2}}}{2} \underset{\lim\limits_{x\longrightarrow 0^-} \frac{1}{x^2} = +\infty}{=}$

$= \frac{1}{2}\left(\lim\limits_{y\longrightarrow +\infty} e^y - \lim\limits_{y\longrightarrow +\infty} e^{-y}\right) = \frac{1}{2}\left(+\infty - 0\right) = +\infty\ .$