Instituto Superior Técnico
Departamento de Matemática
Secção de Álgebra e Análise

Geometria Riemanniana - 1º semestre de 2014/15

Teste de Recuperação 1 - 30/01/2015
Duration: 90 min.
Write down all calculations and relevant justifications

1) Consider the vector fields in $\mathbb{R}^3$

$$X = x\frac{\partial}{\partial x} + y^2\frac{\partial}{\partial y} + z\frac{\partial}{\partial z},\ Y = e^y\frac{\partial}{\partial y},\ Z = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}.$$

(3pt) a) Determine $[Y, Z]$.

(3pt) b) Determine the flow of X.

(3.5pt) c) Can there exist a diffeomorphism $\phi : \mathbb{R}^3 \to \mathbb{R}^3$ such that $D\phi(X) = Z$? Justify your answer.

(3pt) d) Let ψ_t be the flow of X from $b)$. Determine

$$(D\psi_1 Y)_{|(1,1,0)}.$$

(3.5pt) 2) Let M be a smooth manifold and let $X \in \mathcal{X}(M)$ a complete vector field such that all the integral curves of X are closed and, moreover, there exists $T > 0$ such that

$$\gamma_p(t) = \gamma_p(t + T), \forall p \in M, \forall t \in \mathbb{R},$$

where $\gamma_p(t)$ is the integral curve of X through p.

Show that there is an action of S^1 on M such that for every $p \in M$, the orbit of p is

$$O_p = \{\gamma_p(t), t \in \mathbb{R}\}.$$

3) Let M be a compact orientable manifold and let $f_1, \ldots, f_n \in C^\infty(M)$, where $\dim M = n$.

(2pt) a) Show that if $\partial M = \emptyset$ then $df_1 \wedge \cdots \wedge df_n \in \Omega^n(M)$ must have a zero.

(2pt) b) Let $\partial M = A$. Show that if there exists i, with $1 \leq i \leq n$, such that the restriction of f_i to A is constant then

$$\int_M df_1 \wedge \cdots \wedge df_n = 0.$$