Avoiding closed timelike curves with a collapsing rotating null dust shell

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Introduction

• Several solutions to EFE display CTCs: van Stockum, Gödel, NUT, Gott (usually associated to rotation).

• Cosmic censorship implies that for reasonable matter one cannot evolve CTCs from generic initial data, since maximal development is globally hyperbolic and generically inextendible.

• Example: what happens if one tries to create a spinning cosmic string from an incoming rotating cylindrical null shell?
Hyperboloid in Minkowski space

- Parameterize hyperboloid $-\tau^2 + \xi^2 + \eta^2 = a^2$ in Minkowski space $g^- = -d\tau^2 + d\xi^2 + d\eta^2$ by

\[
\begin{align*}
\tau &= u \\
\xi &= a\cos\psi - u\sin\psi \\
\eta &= a\sin\psi + u\cos\psi
\end{align*}
\]

- Induced metric is

\[
h^- = 2a\,du\,d\psi + (u^2 + a^2)\,d\psi^2
\]
Rotating cosmic string

- Rotating cosmic string is given by the metric

\[ g^+ = -(dt + m d\phi)^2 + C^2 dr^2 + r^2 d\phi^2 \]

It is just flat space with unusual identifications.

- CTCs for \( r^2 < m^2 \).

- Mass and angular momentum per unit length are

\[ \mu = \frac{C - 1}{4C} \quad J = \frac{m}{4} \]
Hyperboloid in rotating cosmic string metric

- **Null geodesics** are parameterized by

\[
\begin{align*}
  t &= bC \tan \lambda - mC\lambda \\
  r &= b \sec \lambda \\
  \varphi &= C\lambda + \psi
\end{align*}
\]

- In the new set coordinates \(\{\lambda, b, \psi\}\) the metric becomes

\[
g^+ = 2bC(b-m) \sec^2 \lambda \, d\lambda \, d\psi + C^2 \, db^2 - 2mC \tan \lambda \, db \, d\psi + (b^2 \sec^2 \lambda - m^2) \, d\psi^2
\]
• The metric induced on a hypersurface \( \{b = \text{constant}\} \) is

\[
h^+ = 2bC(b - m) \sec^2 \lambda d\lambda d\psi + (b^2 \sec^2 \lambda - m^2) d\psi^2
\]

or setting \( u = b \tan \lambda \)

\[
h^+ = 2C(b - m) du d\psi + (u^2 + b^2 - m^2) d\psi^2
\]

• Matching conditions are

\[
\begin{cases}
a = C(b - m) \\
a^2 = b^2 - m^2
\end{cases}
\]

In particular that the matching requires \( b > m \), so that the shell \textbf{bounces} before \textbf{CTCs} are revealed in the exterior.
Shell matter

- Computing the jump on the second fundamental form one arrives at
  \[
  T^\alpha\beta \partial_\alpha \otimes \partial_\beta = \frac{m}{8\pi C a \rho} \delta \left( \rho - \sqrt{\tau^2 + a^2} \right) \frac{\partial}{\partial u} \otimes \frac{\partial}{\partial u}
  \]
  where \( \rho^2 = \xi^2 + \eta^2 \).

- Therefore matter is a null dust with surface density
  \[
  \sigma = \frac{m}{8\pi a C \rho} = \frac{C^2 - 1}{16\pi C^2 \rho} = \frac{(C + 1)}{2C} \frac{\mu}{2\pi \rho}
  \]
Similarly
\[ -T^{\alpha\beta} \left( \frac{\partial}{\partial \tau} \right)_\alpha \left( \frac{\partial}{\partial \varphi} \right)_\beta = a\sigma \delta \left( \rho - \sqrt{\tau^2 + a^2} \right) \]
corresponding to a surface angular momentum density
\[ j = a\sigma = \frac{m}{8\pi C \rho} = \frac{1}{C} \frac{J}{2\pi \rho} \]

Densities slightly puzzling. Notice that solution is not stationary (so one cannot use Komar integrals).