Greybody Factors for *d*-Dimensional Black Holes

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(based on work with Troels Harmark and Ricardo Schiappa)

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Talk at Universidade do Porto, 2008

Outline

1. Introduction
   - What are Greybody Factors?
   - How to compute them?
   - Why do we care?
2. Low Frequency
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   - Results
     - Asymptotically Flat Solutions
     - Asymptotically de Sitter Solutions
     - Asymptotically Anti de Sitter Solutions
3. High Imaginary Frequency
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What are Greybody Factors?

- Consider the massless wave equation

$$\Box\Phi = 0$$

on a d-dimensional spherically symmetric black hole background:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{d-2}{}^2.$$

- Here

$$f(r) = 1 - \frac{2\mu}{r^{d-3}} + \frac{\theta^2}{r^{2d-6}} - \lambda r^2$$

and

$$M = \frac{(d-2)\,\Omega_{d-2}}{8\pi G_d}\mu, \qquad \Omega_n = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma\left(\frac{n+1}{2}\right)},$$

$$Q^2 = \frac{(d-2)\,(d-3)}{8\pi G_d}\theta^2,$$

$$\Lambda = \frac{1}{2}(d-1)\,(d-2)\,\Lambda.$$

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What are Greybody Factors?

- Decompose
$$\Phi(t,r,\Omega) = e^{i\omega t}\Phi_{\omega,\ell}(r)Y_{\ell m}(\Omega)$$
- Define the tortoise coordinate x through
$$dx = \frac{dr}{f(r)}$$
- Then wave equation is written as
$$\left[\frac{d^2}{dx^2} + \omega^2 - V(r)\right]\left(r^{\frac{d-2}{2}}\Phi_{\omega,\ell}\right) = 0$$
- Here
$$V(r) = f(r)\left(\frac{\ell(\ell+d-3)}{r^2} + \frac{(d-2)(d-4)f(r)}{4r^2} + \frac{(d-2)f'(r)}{2r}\right)$$

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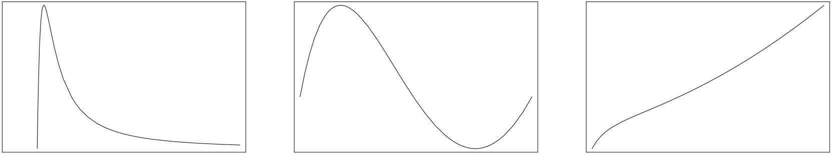
- Potentials for $d = 6$ and $\ell = 0$ in Schwarzschild, Schwarzschild-de Sitter and Schwarzschild-Anti de Sitter:



What are Greybody Factors?

- Choose a solution Φ representing an incoming wave at infinity. Then the greybody factor is

$$\gamma(\omega,\ell) = \frac{\text{Total flux of } \Phi \text{ at the horizon}}{\text{Total flux of } \Phi \text{ at infinity}}$$

- Can interchange "ingoing" $\leftrightarrow$ "outgoing" and "horizon" $\leftrightarrow$ "infinity".
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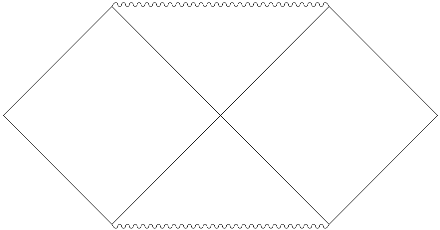

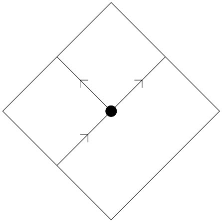

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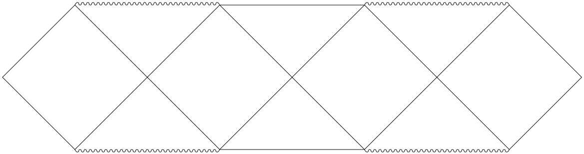

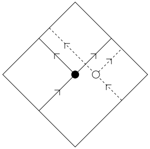

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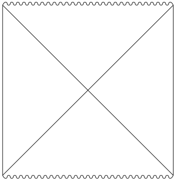

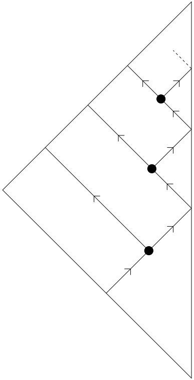

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How to compute them?

- Solve

$$\left[\frac{d^2}{dx^2} + \omega^2 - V(r(x))\right]\Psi_{\omega,\ell} = 0$$

subject to

$$\begin{cases} \Psi_{\omega,\ell} \sim e^{i\omega x} + Re^{-i\omega x}, & x \to +\infty \\ \Psi_{\omega,\ell} \sim Te^{i\omega x}, & x \to -\infty \end{cases}$$

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- Hawking radiation: expectation value $\langle n(\omega) \rangle$ for the number of particles of a given species, emitted in a mode with frequency ω, is given by

$$\langle n(\omega) \rangle = \frac{\gamma(\omega)}{e^{\frac{\omega}{T_H}} \pm 1}$$

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Method

- Split up spacetime into 3 regions:
  - Region I: The region near the (outer) event horizon, defined by $r \simeq R_H$ and $V(r) \ll \omega^2$.
  - Region II: The intermediate region, defined by $V(r) \gg \omega^2$.
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Results

Asymptotically Flat Solutions

- $\gamma(\omega) = \frac{4\pi\omega^{d-2}R_H^{d-2}}{2^{d-2}[\Gamma(\frac{d-1}{2})]^2}$ (R_H = radius of the (outer) horizon).
- Results best described in terms of the absorption cross section:

$$\sigma(\omega) = \gamma(\omega)|\alpha|^2$$

where α is the coefficient of the $\ell = 0$ term in the decomposition of a plane wave into ingoing spherical harmonic waves.
- We have the universal result

$$\sigma(\omega) = A_H$$

where A_H is the area of the (outer) horizon.
- First done in [Das, Gibbons and Mathur].

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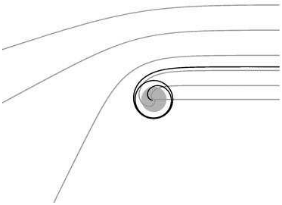
- For comparison, in the high frequency limit we have the geometric optics result

$$\sigma(\omega) = \nu(d) A_H$$

where

$$\nu(d) = \frac{1}{d-2}\left(\frac{d-1}{2}\right)^{\frac{d-2}{d-3}}\left(\frac{d-1}{d-3}\right)^{\frac{d-2}{2}}\frac{\Omega_{d-3}}{\Omega_{d-2}} \qquad \left(\nu(4) = \frac{27}{8}, \nu(5) = \frac{16}{3\pi}, \dots\right)$$



Results

Asymptotically de Sitter Solutions

- $\gamma(\omega) = 4h(\hat{\omega})\frac{A_H}{A_C}$.
- A_H = area of the black hole horizon
  A_C = area of the cosmological horizon
  $\hat{\omega}$ = frequency normalized by the cosmological radius
  $h(\hat{\omega})$ given by

$$h(\hat{\omega}) = \prod_{n=1}^{\frac{d-2}{2}} \left(1 + \frac{\hat{\omega}^2}{(2n-1)^2}\right)$$

for even $d \geq 4$ and

$$h(\hat{\omega}) = \frac{\pi\hat{\omega}}{2} \coth\left(\frac{\pi\hat{\omega}}{2}\right) \prod_{n=1}^{\frac{d-3}{2}} \left(1 + \frac{\hat{\omega}^2}{(2n)^2}\right)$$

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Asymptotically de Sitter Solutions

- Small black holes (R_H much smaller than the cosmological scale).
- $\gamma(\hat{\omega}) = 1 - \left| \frac{1-z(\hat{\omega})}{1+z(\hat{\omega})} \right|^2$ where

$$z(\hat{\omega}) = \frac{\pi}{2^{d-2}} \frac{[\Gamma(\frac{d-1}{2})]^2}{[\Gamma(\frac{d-1+\hat{\omega}}{2})\Gamma(\frac{d-1-\hat{\omega}}{2})]^2} \frac{\hat{\omega}^{d-2}}{(\kappa R_H)^{d-2}}$$

- Again universal result.
- $\gamma(\hat{\omega}) = 0$ for $\hat{\omega} = d - 1 + 2n$ with $n \in \{0, 1, 2, ...\}$. These are exactly the normal frequencies for pure AdS.
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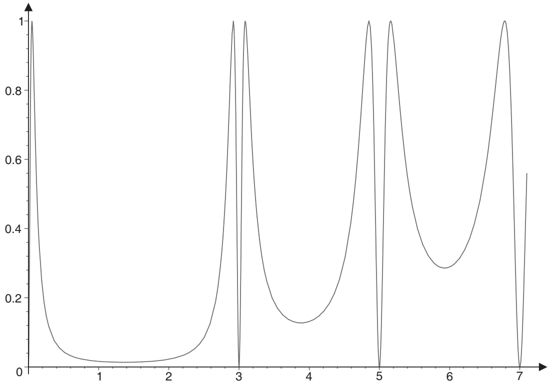

Outline

Method

- Complex ω:
  - Φ_ω^* is replaced by $\Phi_{-\omega}$.
  - R^*, T^* are replaced by $\widetilde{R}$, $\widetilde{T}$.
  - $R\widetilde{R} + T\widetilde{T} = 1$.
- $\omega \to i\infty$: solve equation in the complex plane at origin, horizons, infinity; match solutions along (anti-)Stokes lines $\operatorname{Re} x = 0$ and use monodromies of $\Phi_\omega, \Phi_{-\omega}$ at horizons to implement boundary conditions.

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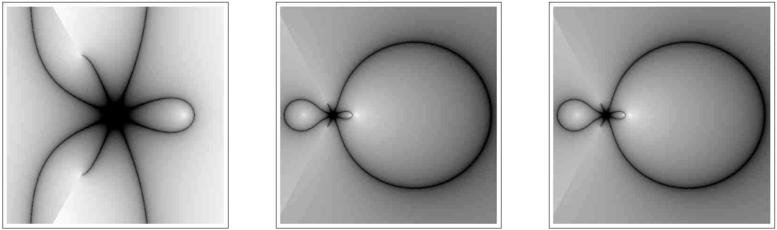

Method

- Considered gravitational perturbations (tensor, vector, scalar) and any ℓ (includes massless scalar field).
- Potentials are slightly more complicated: for example

$$V_S(r) = \frac{f(r)U(r)}{16r^2H^2(r)},$$

where

$$H(r) = \ell\left(\ell + d - 3\right) - \left(d - 2\right) + \frac{\left(d-1\right)\left(d-2\right)\mu}{r^{d-3}},$$

and

$$\begin{aligned} U(r) \quad = \quad & -\Big[4d\left(d-1\right)^2\left(d-2\right)^3\frac{\mu^2}{r^{2d-6}} - 24\left(d-1\right)\left(d-2\right)^2\left(d-4\right)\Big\{\ell\left(\ell+d-3\right)-\left(d-2\right)\Big\}\frac{\mu}{r^{d-3}} + \\ & +4\left(d-4\right)\left(d-6\right)\Big\{\ell\left(\ell+d-3\right)-\left(d-2\right)\Big\}^2\Big]\lambda r^2 + 8\left(d-1\right)^2\left(d-2\right)^4\frac{\mu^3}{r^{3d-9}} + 4\left(d-1\right)\left(d-2\right)\cdot \\ & \cdot\Big[4\left(2d^2-11d+18\right)\Big\{\ell\left(\ell+d-3\right)-\left(d-2\right)\Big\} + \left(d-1\right)\left(d-2\right)\left(d-4\right)\left(d-6\right)\Big]\frac{\mu^2}{r^{2d-6}} - 24\left(d - \right. \\ & \cdot\Big[\left(d-6\right)\Big\{\ell\left(\ell+d-3\right)-\left(d-2\right)\Big\} + \left(d-1\right)\left(d-2\right)\left(d-4\right)\Big]\Big\{\ell\left(\ell+d-3\right)-\left(d-2\right)\Big\}\frac{\mu}{r^{d-3}} + \\ & +16\Big\{\ell\left(\ell+d-3\right)-\left(d-2\right)\Big\}^3 + 4d\left(d-2\right)\Big\{\ell\left(\ell+d-3\right)-\left(d-2\right)\Big\}^2. \end{aligned}$$

Method

- Considered gravitational perturbations (tensor, vector, scalar) and any ℓ (includes massless scalar field).
- Potentials are slightly more complicated: for example

$$V_S(r) = \frac{f(r)U(r)}{16r^2H^2(r)},$$

where

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Outline

Results

Asymptotically Flat Solutions

- Schwarzschild:

$$\gamma(\omega) = T(\omega)\widetilde{T}(\omega) = \frac{e^{\frac{\omega}{T_H}} - 1}{e^{\frac{\omega}{T_H}} + 3}$$

where T_H is the Hawking temperature of the event horizon.

- Poles are the quasinormal frequencies.
- First done in [Neitzke].
- Exotic statistics?

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Results

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- Reissner-Nordstrom:

$$\gamma(\omega) = T(\omega)\widetilde{T}(\omega) = \frac{e^{\frac{\omega}{T_H^+}} - 1}{e^{\frac{\omega}{T_H^+}} + (1 + 2\cos(\pi j)) + (2 + 2\cos(\pi j))e^{-\frac{\omega}{T_H^-}}}$$

where $j = \frac{d-3}{2d-5}$ for tensor and scalar type perturbations, and $j = \frac{3d-7}{2d-5}$ for vector type perturbations.

Results

Asymptotically de Sitter Solutions

- Schwarzschild-de Sitter:

$$\gamma(\omega) = \frac{-2\sinh\left(\frac{\pi\omega}{k_H}\right)\sinh\left(\frac{\pi\omega}{k_C}\right)}{3\cosh\left(\frac{\pi\omega}{k_H}+\frac{\pi\omega}{k_C}\right)+\cosh\left(\frac{\pi\omega}{k_H}-\frac{\pi\omega}{k_C}\right)}$$

where k_H, k_C are the surface gravities at the horizons ($k_C < 0$).

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Asymptotically Anti de Sitter Solutions

- Schwarzschild-Anti de Sitter: $\gamma(\omega) = 1$.
- Reissner-Nordstrom-Anti de Sitter: $\gamma(\omega) = 1$.
- In this case there are no poles because of reflecting boundary conditions are imposed when computing quasinormal modes.

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Summary and Outlook

- Computed greybody factors for spherically symmetric bacgrounds in the two regimes:
  - Massless scalar waves, $\ell = 0$, at low (real) frequency;
  - Gravitational perturbations (includes massless scalar waves), any ℓ, at large imaginary frequencies.
- Found universal behaviour at low frequencies.
- Hints of exotic statistics at high imaginary frequencies do not seem to extend beyond Schwarzschild.
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