Cosmic no-hair in spherically symmetric black hole spacetimes

José Natário
(joint with João L. Costa and Pedro Oliveira)

CAMGSD – Instituto Superior Técnico
Universidade de Lisboa
(GPSEinstein) PTDC/MAT-ANA/1275/2014 , UID/MAT/04459/2013.

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Positive cosmological constant $\Lambda > 0$

- Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 2 T_{\mu\nu}$$

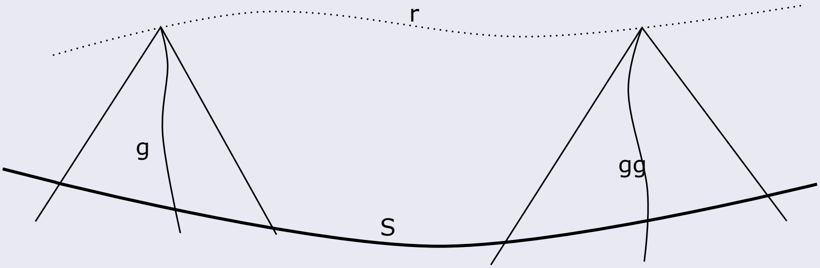
- $\Lambda > 0$ introduced by Einstein in 1917 to obtain a static cosmological solution; later dismissed it as his “biggest blunder” in view of Hubble’s galaxy redshift observations.
- Made a triumphant comeback into the standard model of cosmology in 1998 due to the observation of the accelerated expansion of the Universe.
- For late cosmological times, $\Lambda > 0$ is expected to completely dominate the dynamics, damping all inhomogeneities and anisotropies.

Cosmic no-hair conjecture

- **Cosmic no-hair conjecture** (Gibbons and Hawking, 1977): Generic expanding solutions of Einstein's field equations with a positive cosmological constant approach the de Sitter solution asymptotically.
- As usual in General Relativity, obtaining a precise statement for this conjecture is one its biggest challenges.
- The statement above is sufficiently vague to accomodate a considerable amount of relevant work: Wald, Friedrich, Rendall-Tchapnda, Rodnianski-Speck, Lubbe-Kroon, Speck, Ringström, Dafermos-Rendall, Vasy, Alho-Costa-Natário, Fajman-Kröncke, Gasperin-Kroon, Oliynyk, Schlue, Andréasson-Ringström, ...

Cosmic silence

- A **naïve, but hopeless**, attempt is to assert that cosmic no-hair holds if generic solutions contain foliations of a neighborhood of future null infinity $\mathcal{I}^+$ along which they approach de Sitter uniformly.
- This attempt is doomed to fail as a consequence of **cosmic silence**.



- Fundamental new insight: *all observers will see (for late cosmological times) the spacetime structure around them approach that of a de Sitter spacetime. However, this will not happen uniformly for all observers.*
- Here we will follow the spirit, although not the letter, of the Andréasson-Ringström formulation.

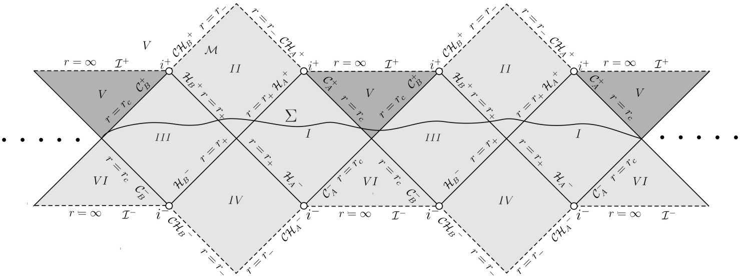
Cosmic no-hair in black hole space-times

Either by symmetry assumptions or smallness conditions, all the existing (non-linear) results exclude the existence of black holes a priori. Partial exceptions:

- Dafermos-Rendall (2016) established the stability of the Penrose diagram for the expanding region of spherically symmetric solutions of the Einstein-Vlasov system. This is in line with the expectations of cosmic no-hair but does not provide enough quantitative information to show that the geometry is approaching that of de Sitter.
- Gasperin-Kroon (2015) obtained a "partial" stability result of the expanding region of Schwarzschild-de Sitter (SdS), without symmetry assumptions.
- Schlue (2017) made considerable progress towards the proof of the (non-linear) stability of the expanding region of SdS for the (vacuum) Einstein equations, without symmetry assumptions.

Reissner-Nordström-de Sitter (RNdS)

Subextremal members of the RNdS family model static, spherically symmetric and electrically charged black holes in an expanding universe. They are solutions of the Einstein-Maxwell equations parameterized by mass M, charge e and cosmological constant Λ.



Einstein-Maxwell-scalar field system with a cosmological constant

The equations for a self-gravitating massless scalar field in a sourceless electromagnetic field with a cosmological constant are

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 2T_{\mu\nu}$$
$$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}\partial_\alpha\phi\,\partial^\alpha\phi\,g_{\mu\nu} + F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}g_{\mu\nu}$$
$$\Box_g\phi = 0$$
$$dF = d^\star F = 0$$

Spherical symmetry

- By spherical symmetry we mean that the metric takes the form

$$g = -\Omega^2(u,v)du\,dv + r^2(u,v)\left(d\theta^2 + \sin^2\theta\,d\varphi^2\right)$$

with the scalar field and the Faraday electromagnetic 2-form satisfying

$$\phi = \phi(u,v)$$
$$F = -E(u,v)\frac{\Omega^2(u,v)}{2}du \wedge dv$$

- In this setting, the electromagnetic field completely decouples:

$$^\star F = E(u,v)r^2(u,v)\sin\theta\,d\theta \wedge d\varphi \;\Rightarrow\; E(u,v) = \frac{e}{r^2(u,v)}$$

Einstein-Maxwell-scalar field system in spherical symmetry

- Introduce the renormalized Hawking mass ϖ through

$$1-\frac{2\varpi}{r}+\frac{e^2}{r^2}-\frac{\Lambda}{3}r^2=-\frac{4\partial_u r\partial_v r}{\Omega^2}$$

The Einstein-Maxwell-scalar field equations become

$$\partial_u\partial_v\phi=-\frac{\partial_u r\,\partial_v\phi}{r}-\frac{\partial_v r\,\partial_u\phi}{r}$$

$$\partial_u\partial_v r=\partial_u r\,\partial_v r\,\frac{\frac{2\varpi}{r^2}-\frac{2e^2}{r^3}-\frac{2\Lambda}{3}r}{1-\frac{2\varpi}{r}+\frac{e^2}{r^2}-\frac{\Lambda}{3}r^2}$$

$$\partial_u\varpi=\left(1-\frac{2\varpi}{r}+\frac{e^2}{r^2}-\frac{\Lambda}{3}r^2\right)\frac{(r\partial_u\phi)^2}{2\partial_u r}$$

$$\partial_v\varpi=\left(1-\frac{2\varpi}{r}+\frac{e^2}{r^2}-\frac{\Lambda}{3}r^2\right)\frac{(r\partial_v\phi)^2}{2\partial_v r}$$

Our main result

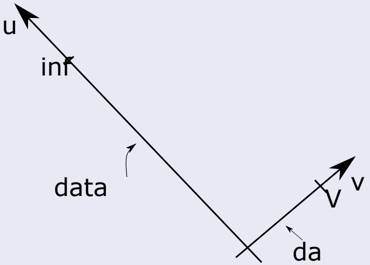
Fix, as a reference solution, a subextremal member of the RNdS family with parameters (M, e, Λ) and cosmological radius r_c, and assume that:

(i) the radius of symmetry satisfies $r(u, 0) \to r_c$ as $u \to \infty$;

(ii) the (renormalized) Hawking mass satisfies $\varpi(u, 0) \to M$ as $u \to \infty$;

(iii) $\partial_v r(u, 0) > 0$, for all $u \geq 0$, and

(iv) $|\partial_u \phi(u, 0)| \leq C$, for some $C > 0$ and all $u \geq 0$.



Our main result

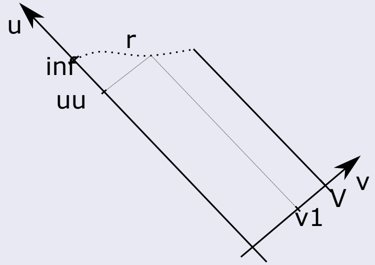
Then, there exist $U, V > 0$ such that, in

$$\mathcal{Q} = \tilde{\mathcal{Q}} \cap [U, \infty) \times [0, V),$$

we have:

- Blow up of the radius function: for all $v_1 \in (0, V)$, there exists $u^*(v_1) < \infty$ such that

$$r(u, v) \to \infty \text{ as } (u, v) \to (u^*(v_1), v_1) .$$



Our main result

- Asymptotic behavior of the scalar field: there exists $C > 0$ such that
$$\left|\frac{\partial_v \phi}{\partial_v r}\right| + \left|\frac{\partial_u \phi}{\partial_u r}\right| \leq C r^{-2} \, .$$

- $r = \infty$ is spacelike: for a parameterization of the curves of constant r of the form $v \mapsto (u_r(v), v)$ there exists a constant $C_1 > 0$ and a continuous function $(0, V] \ni v \mapsto C_2(v) \in \mathbb{R}^+$, which may blow up as $v \to 0$, such that
$$-C_2(v) < u_r'(v) < -C_1$$
holds for all $r > r_c$ and all $v > 0$.

Our main result

- Cosmic no-hair:
  - Let γ be a causal curve along which r is unbounded.
  - Let i_{dS} be a point in the future null infinity of the de Sitter spacetime with cosmological constant Λ. Let $\{e_I\}_{I=0,1,2,3}$ be an orthonormal frame in de Sitter defined on (the causal past) $J^-(i_{dS})$.

  Then there exists a diffeomorphism mapping $J^-(\gamma) \cap \{r > r_1\}$ to a neighborhood of i_{dS} in $J^-(i_{dS})$ such that, for $r_2 \geq r_1$,

$$\sup_{J^-(\gamma)\cap\{r\geq r_2\}} |g_{IJ} - {}^{dS}g_{IJ}| \lesssim r_2^{-2} ,$$

  and

$$\sup_{J^-(\gamma)\cap\{r\geq r_2\}} |R^I_{JKL} - {}^{dS}R^I_{JKL}| \lesssim r_2^{-2} .$$