The location of trapped surfaces

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Outline

References

- I Bengtsson and JMMS, *Note on trapped surfaces in the Vaidya solution*, Phys. Rev. D **79** 024027 (2009) [arXiv:0809.2213]
- I Bengtsson and JMMS, *The boundary of the region with trapped surfaces in spherical symmetry*, Preprint
- JMMS, *On the boundary of the region containing trapped surfaces*, AIP Conf. Proc. **1122** 72 (2009) [arXiv:0812.2767]

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First fundamental form:

$$\gamma_{AB}(\lambda) = g|_S(\vec{e}_A, \vec{e}_B) = g_{\mu\nu}(\Phi) e^\mu_A e^\nu_B$$

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Then, $\forall x \in S$

$$T_x\mathcal{V} = T_xS \oplus T_xS^\perp$$

called *tangent* and *normal* parts.

Notation: extrinsic curvature

In particular, we have $\nabla_{\vec{e}_A}\vec{e}_B = \overline{\Gamma}^C_{AB}\vec{e}_C - \vec{K}_{AB}$

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OBS: $\vec{\boldsymbol{K}}$ is orthogonal to S. $\vec{\boldsymbol{K}} : \mathfrak{X}(S) \times \mathfrak{X}(S) \longrightarrow \mathfrak{X}(S)^{\perp}$ where $\mathfrak{X}(S)$ ($\mathfrak{X}(S)^{\perp}$) is the set of vector fields tangent (perpendicular) to S.

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The second fundamental form of S in $(\mathcal{V}, g)$

relative to any $\vec{n} \in \mathfrak{X}(S)^{\perp}$ is:

$$K_{AB}(\vec{n}) \equiv n_\mu K^\mu_{AB} \,.$$

These are 2-covariant symmetric tensor fields on S.

Mean curvature vector. Null expansions

For a spacelike surface there are two *independent* normal vector fields, we can choose them to be future-pointing and null, $\vec{k}^{\pm} \in \mathfrak{X}(S)^{\perp}$; adding $k_{+\mu}k_{-}^{\mu} = -1$, there remains the freedom

$$\vec{k}^{+} \longrightarrow \vec{k}'^{+} = \sigma^{2}\vec{k}, \qquad \vec{k}^{-} \longrightarrow \vec{k}'^{-} = \sigma^{-2}\vec{k}^{-}$$

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$\theta^{\pm} \equiv \gamma^{AB}K_{AB}(\vec{k}^{\pm})$ are called the future null expansions

Future-trapped surfaces: $\vec{H}$ is future on S

The main cases are:

$\vec{H}$	Expansions	Type of surface
zero	$\theta^+ = \theta^- = 0$	stationary or minimal
null and future	$\theta^+ = 0, \theta^- < 0$	marginally f-trapped
null and future	$\theta^+ < 0, \theta^- = 0$	marginally f-trapped
timelike future	$\theta^+ < 0, \theta^- < 0$	f-trapped

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Schwarzchild (in units with $G = c = 1$)

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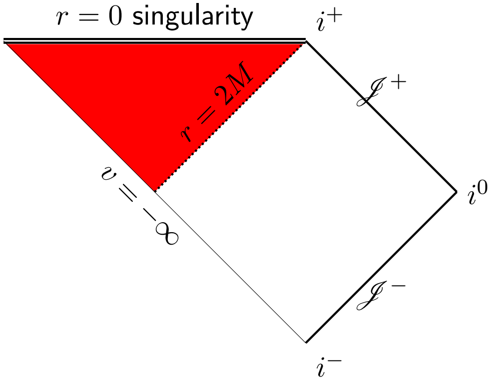
The Event Horizon

EH: $\qquad r - 2M = 0$

The Penrose, or conformal, diagram



The event horizon (EH) in general

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The Event Horizon EH

The boundary of the past of $\mathscr{J}^{+}$.
By definition, this is always a null hypersurface.

Dynamical situations: Evolving Black Holes (BH)

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- If not, what does?
- for instance, where does the Hawking radiation comes from?

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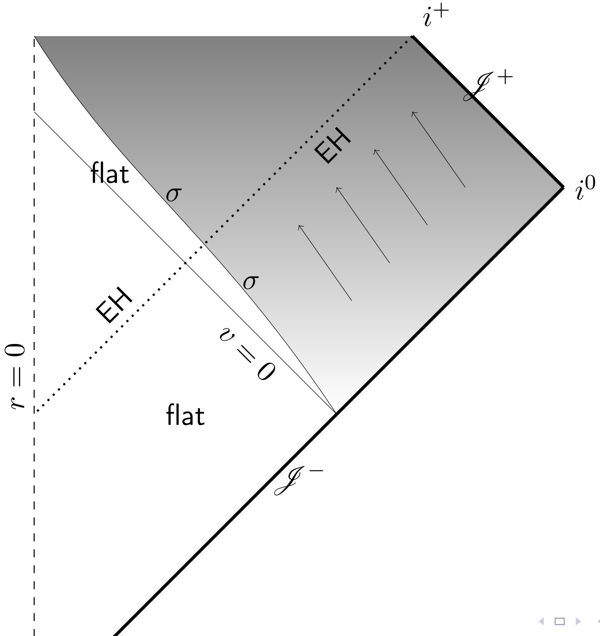
- Problem with the Event Horizon (EH): teleology.

A dynamical situation



MTT and dynamical horizons

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If they are spacelike, they are called dynamical horizons.
In spherical symmetry, there are privileged spherically symmetric MTTs. These are foliated by marginally f-trapped round spheres, and invariantly defined by

$$g^{-1}(dr, dr) = 0$$

where r is the area coordinate.

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- Dynamical or Future Outer Trapping Horizons? They are not unique!

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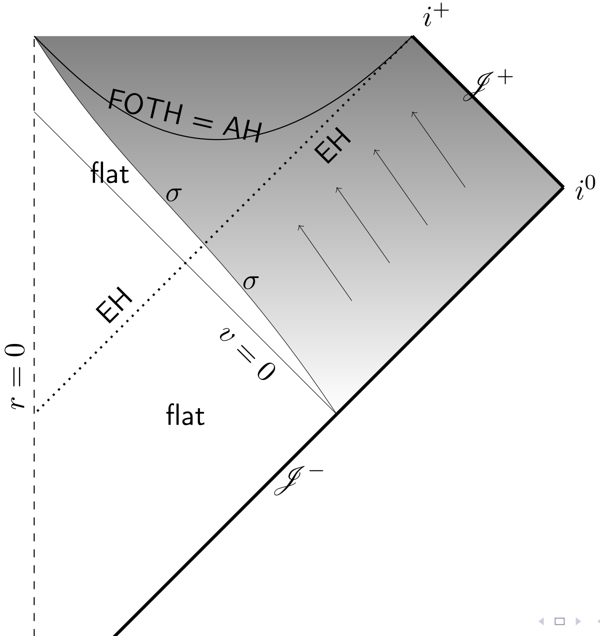

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- What is the surface, or the boundary, of an evolving black hole?
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- Can closed trapped surfaces penetrate outside *all* dynamical horizons?

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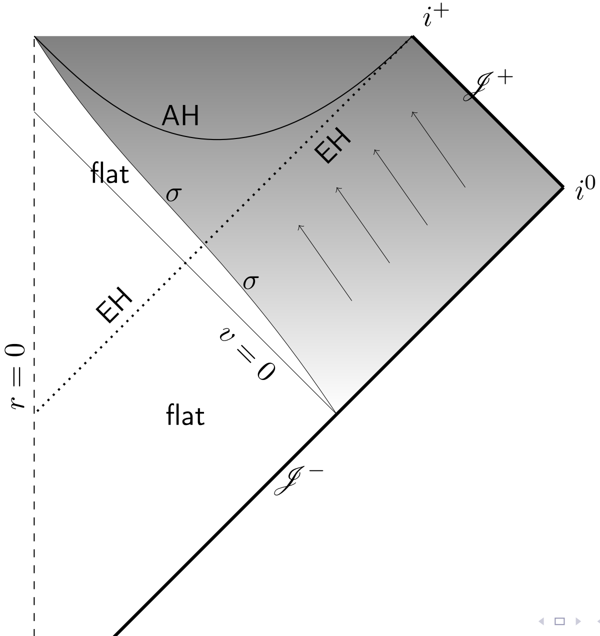
- Problem with the Event Horizon (EH): teleology.
- Dynamical or Future Outer Trapping Horizons? (Spacelike hypersurfaces foliated by closed marginally trapped surfaces; example: the apparent 3-horizon AH). They are not unique!
- Can closed trapped surfaces penetrate outside *all* dynamical horizons?
- Can closed trapped surfaces actually penetrate into flat regions?
- Where can there be closed trapped surfaces?
- In summary, where is the red region now?

A dynamical situation



The future-trapped region $\mathscr{T}$ and its boundary $\mathscr{B}$

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The boundary $\mathscr{B}$

We denote by $\mathscr{B}$ the boundary of the future trapped region $\mathscr{T}$:

$$\mathscr{B} \equiv \partial \mathscr{T}$$

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$\mathscr{T}$ is not necessarily connected.

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- The location of $\mathscr{B}$ provides important physical information due to the fundamental relevance of closed trapped surfaces in the development of black holes and singularities.
- More importantly, it provides a precise limit as to where dynamical horizons or marginally trapped tubes can develop.
- In a way, $\mathscr{B}$ is the true genesis of everything that is distinctive of black holes: quasi-local horizons and f-trapped closed surfaces.

The boundary $\mathscr{B}$ in spherical symmetry

Result ($\mathscr{B}$ has spherical symmetry)

In arbitrary spherically symmetric spacetimes, $\mathscr{T}$ and $\mathscr{B}$ have spherical symmetry.

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Result

In arbitrary spherically symmetric spacetimes, $\mathscr{B}$ (if not empty) is a spherically symmetric hypersurface without boundary.

Spherically symmetric spacetimes

In advanced coordinates

$$ds^2 = -e^{2\beta}\left(1 - \frac{2m(v,r)}{r}\right)dv^2 + 2e^{\beta}dvdr + r^2 d\Omega^2$$

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- Future-pointing radial null geodesic vector fields

$$\vec{\ell} = -e^{-\beta}\partial_r, \qquad \vec{k} = \partial_v + \frac{1}{2}\left(1 - \frac{2m}{r}\right)e^{\beta}\partial_r$$

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Result ($\mathscr{B}$ is not a spherically symmetric MTT)

In arbitrary spherically symmetric spacetimes, $\mathscr{B}$ never coincides with any spherically symmetric MTT if $G_{\mu\nu}k^\mu k^\nu \neq 0$ on the MTT.

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The result and a sketch of its proof were communicated to us by R.M. Wald, who informed us that they arose in conversations with G. Galloway.

Proof

- Take the MTT and set $\vec{k}^- = \vec{\ell}$ and $\vec{k}^+ = \vec{k}$, so that he null expansions on the round spheres in MTT are

$$\theta^- = -e^{-\beta}\frac{2}{r} < 0, \qquad \theta^+ = 0\,.$$

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- Perturb any marginally f-trapped 2-sphere $\varsigma \in$ MTT along a direction $f\vec{n}$ orthogonal to ς, where f is a function on ς and

$$\vec{n} = -\vec{\ell} + \frac{n_\mu n^\mu}{2}\vec{k}\Big|_\varsigma \qquad k_\mu n^\mu = 1.$$

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- The variation of the vanishing expansion $\theta^+ = 0$ reduces to

$$\delta_{f\vec{n}}\theta^+ = -\Delta_\varsigma f + f\left(\frac{1}{r^2} - G_{\mu\nu}k^\mu \ell^\nu - \frac{n_\rho n^\rho}{2}G_{\mu\nu}k^\mu k^\nu\right)\Big|_\varsigma$$

Here Δ_ς is the Laplacian on ς.

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$$\delta_{f\vec{n}}\theta^+ = a_0 \left(\frac{1}{r^2} - G_{\mu\nu}k^\mu \ell^\nu - \frac{n_\rho n^\rho}{2} G_{\mu\nu}k^\mu k^\nu \right)\Big|_\varsigma$$

$$+a_N \left(\frac{1+N(N+1)}{r^2} - G_{\mu\nu}k^\mu \ell^\nu - \frac{n_\rho n^\rho}{2} G_{\mu\nu}k^\mu k^\nu \right)\Big|_\varsigma P_N(\cos\theta).$$

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- The red summand can be made to vanish by choosing

$$n_\rho n^\rho = \frac{2}{G_{\mu\nu}k^\mu k^\nu|_\varsigma} \left(\frac{1+N(N+1)}{r^2} - G_{\mu\nu}k^\mu \ell^\nu \right)\Big|_\varsigma .$$

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- With this choice $\delta_{f\vec{n}}\theta^+ = -a_0 \frac{N(N+1)}{r_\varsigma^2}$. Hence, the deformed surface is f-trapped — for small enough $a_0 > 0$.

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  Using $\Delta_\varsigma P_l = -\frac{l(l+1)}{r_\varsigma^2} P_l$, the variation becomes

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$$+ a_N \left(\frac{1 + N(N+1)}{r^2} - G_{\mu\nu}k^\mu \ell^\nu - \frac{n_\rho n^\rho}{2} G_{\mu\nu}k^\mu k^\nu \right)\Big|_\varsigma P_N(\cos\theta).$$

- The red summand can be made to vanish by choosing

$$n_\rho n^\rho = \frac{2}{G_{\mu\nu}k^\mu k^\nu|_\varsigma} \left(\frac{1 + N(N+1)}{r^2} - G_{\mu\nu}k^\mu \ell^\nu \right)\Big|_\varsigma .$$

  (Here is where one needs that $G_{\mu\nu}k^\mu k^\nu|_\varsigma \neq 0$).
- With this choice $\delta_{f\vec{n}}\theta^+ = -a_0 \frac{N(N+1)}{r_\varsigma^2}$. Hence, the deformed surface is f-trapped — for small enough $a_0 > 0$.
- As $f = a_0 + a_N P_N(\cos\theta)$, setting $a_N < -a_0 < 0$ implies that f is negative around $\theta = 0$ and positive where $P_N \leq 0$. Thus, the deformed surface is f-trapped and penetrates both sides of the MTT.

Trapped surfaces penetrate flat portions!

Example: imploding Vaidya spacetime

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$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$

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The apparent 3-horizon: encloses the future-trapped spheres

AH: $\qquad r - 2m(v) = 0$

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- Numerical investigations (E. Schnetter and B. Krishnan (PRD **73** (2006) 021502(R)), however, were incapable of finding closed f-trapped surfaces to the past of the apparent 3-horizon AH.
- We have been able to construct, analitically and explicitly, examples of closed f-trapped surfaces in the (self-similar) Vaidya spacetime.

Closed trapped surfaces penetrating the flat region

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$$m(v) = \begin{cases} 0 & v < 0 \\ \mu v & 0 \leq v \leq M/\mu \\ M & v > \mu \end{cases}$$

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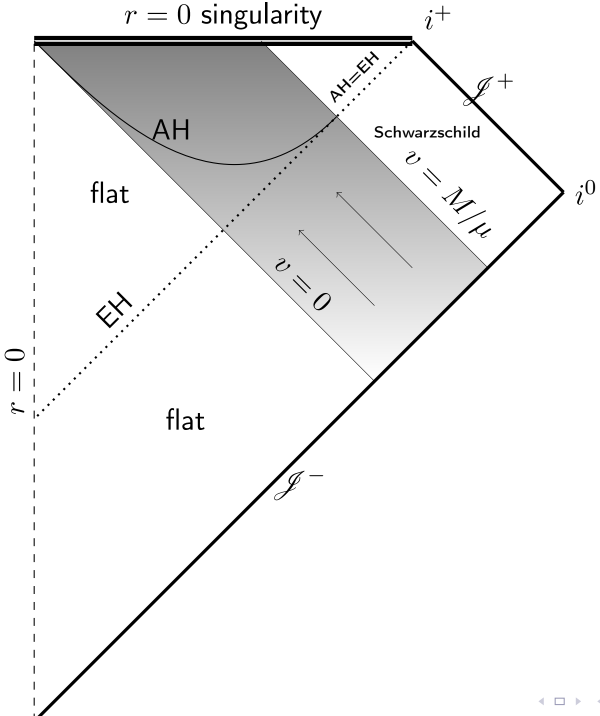
Thus, this is flat for $v < 0$, it ends in a Schwarzschild region with mass M ($v > M/\mu$), and it is self-similar in the intermediate Vaidya region for $0 < v < M/\mu$.



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with $a > b^2/4$ and C =constant. $(dv/dr \to \infty$ at $v = br)$.

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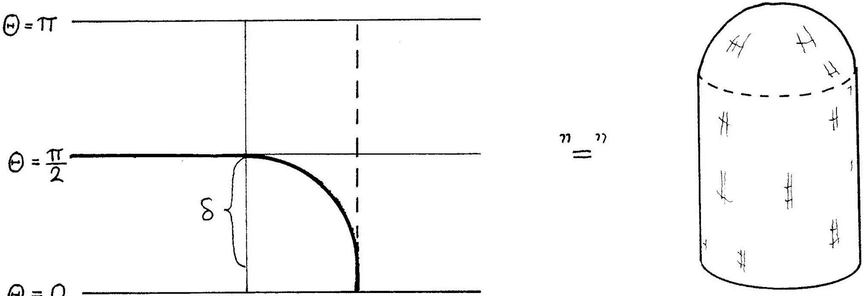
- Schwarzschild region: another disk composed of two parts
  - a cylinder with $\theta = \pi/2; \qquad r = \gamma M$
  - a final "capping" disk defined by

$$\left(\theta - \frac{\pi}{2} + \delta\right)^2 + \left(\frac{v}{\gamma M} - c_1\right)^2 = \delta^2$$

    with constants c_1 and δ.


Θ = π
Θ = π/2
δ
Θ = 0
"="

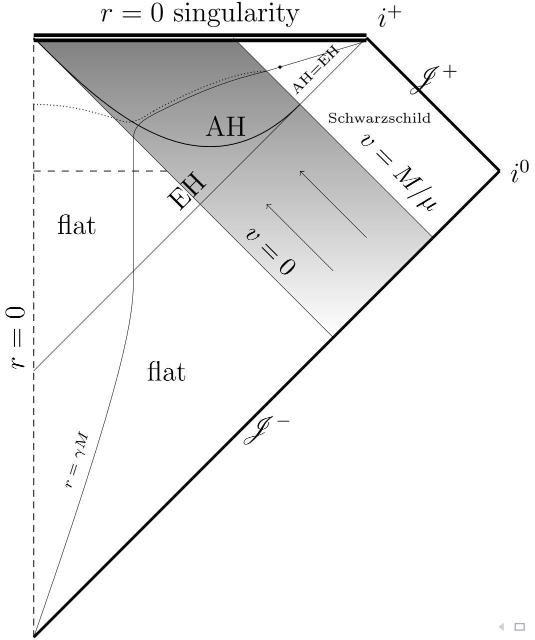
The surface is future-trapped if

1. $t_0 < k, \quad k > 0$
2. $0 < a < b$
3. $1 > \gamma = \frac{1}{b\mu}; \qquad a \geq \frac{1}{\mu}$
4. $0 < \delta \leq \frac{\pi}{2} \qquad \sqrt{\frac{2}{\gamma - 1}}\left(\frac{1}{\gamma} - 1\right) > \frac{1}{\delta}$

Restriction on the mass growth

But these conditions imply in turn a restriction on the mass function:

$$\mu = \frac{1}{\gamma b} > \frac{1}{\gamma}\frac{b}{4a} > \frac{1}{4\gamma} \qquad \gamma < 0.68514$$



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Closed trapped surfaces are highly non-local too

They can have portions in a flat region of spacetime whose *whole* past is also flat in clairvoyance of energy that will cross them elsewhere to make their compactness feasible.

Fundamental general results

$$(\pounds_{\vec{\xi}}\, g)_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

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$$(\pounds_{\vec{\xi}} g|_S)_{\mu\nu}\, e^\mu_A e^\nu_B = \overline{\nabla}_A \overline{\xi}_B + \overline{\nabla}_B \overline{\xi}_A + 2\xi_\mu|_S K^\mu_{AB}$$

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Contracting now with γ^{AB} we arrive at

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Integrating the main formula on S, the divergence term integrates to zero so that

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$\Longrightarrow \vec{H}$ cannot be timelike future pointing all over S

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- $$\xi_{[\mu}\nabla_{\nu}\xi_{\rho]} = 0 \iff \xi_{\mu} = -F\partial_{\mu}\tau$$

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Theorem (No minimum of τ)

- *$\vec{\xi}$ future-pointing and hypersurface-orthogonal on $\mathscr{R} \subset \mathcal{V}$*
- *then, any (marginally) f-trapped surface S cannot have a local minimum of τ at any $q \in \mathscr{R}$ where*

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- $\partial^2\bar{\tau}/\partial\lambda^A\partial\lambda^B|_q$ cannot be positive (semi)-definite. ∎

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Kerr-Schild vector fields

(B. Coll, S.R. Hildebrandt and JMMS, GRG **33** 649 (2001))

$$(\pounds_{\vec{\xi}} g)_{\mu\nu} = 2h\ell_\mu \ell_\nu, \qquad (\pounds_{\vec{\xi}} \ell)_\mu = b\ell_\mu$$

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Furthermore, $\vec{\xi}$ is obviously hypersurface-orthogonal

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Consequently, f-trapped closed surfaces are absent in generic expanding *Robertson-Walker models. They can only be present in models with contracting phases.*

Trapped round spheres in Robertson-Walker

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- As the slices are homogeneous, there are closed trapped spheres through *any* point of the the slices with $\dot{a} \neq 0$ and $\dot{a}^2 + k > 0$.
- These are, of course, past-trapped if $\dot{a} > 0$ and f-trapped if $\dot{a} < 0$.

Trapped round spheres in Robertson-Walker

- It is well-known that there are closed trapped spheres at any $t =$constant slice with $\dot{a} \neq 0$ and $\dot{a}^2 + k > 0$ (this is simply the positivity of the energy density).
- As the slices are homogeneous, there are closed trapped spheres through *any* point of the the slices with $\dot{a} \neq 0$ and $\dot{a}^2 + k > 0$.
- These are, of course, past-trapped if $\dot{a} > 0$ and f-trapped if $\dot{a} < 0$.
- The remaining case of 3-sphere slices $(k = 1)$ with $\dot{a} = 0$ is such that all round 2-spheres in the slice are untrapped except for the equatorial one which is a minimal surface.

The boundary $\mathscr{B}$ for Robertson-Walker metrics

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- *In the case that $\dot{a}$ changes sign, the boundary $\mathscr{B}$ is necessarily contained in the region with $\dot{a} = 0$: $\mathscr{B} \subset \{\dot{a} = 0\}$.*

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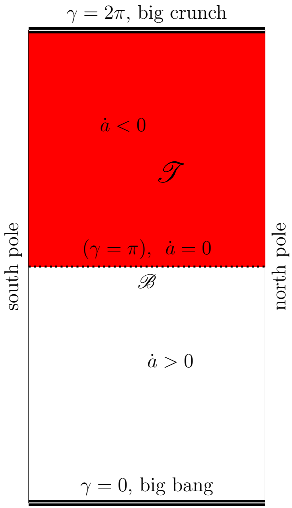
However, not all such hypersurfaces will be part of the boundary in general (e.g., de Sitter, which has $\mathscr{B} = \emptyset$ and $\mathscr{T} = \mathcal{V}$)

The closed ($k = 1$) Fridman dust model



Application to Vaidya: the hypersurface Σ

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Kerr-Schild vector field for Vaidya

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Kerr-Schild vector field for Vaidya

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It is immediate to get

$$(\pounds_{\vec{\xi}} g)_{\mu\nu} = 2\frac{dm}{dv}\ell_\mu \ell_\nu \,, \qquad (\pounds_{\vec{\xi}} \ell)_\mu = 0$$

so that $h = dm/dv \geq 0$ is one of the requirements of previous theorems.

Other requirements on $\vec{\xi}$

- To check the other requirements, note that $\vec{\xi}$ is hypersurface orthogonal, with the level function τ defined by

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Besides, $\vec{\xi}$ is timelike on $\mathscr{R}_0$ and null at the AH: $r = 2m(v)$.

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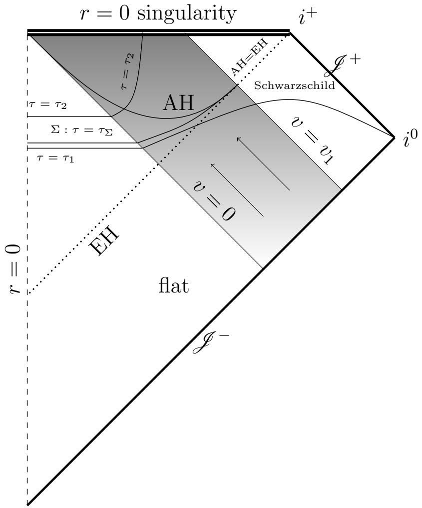
The location of Σ depends on whether $\frac{dm}{dv}(0) > 1/8$ or not. In the former case, Σ does penetrate the flat region. It may not be so in the other cases. (For the self-similar Vaidya case previously considered, Σ penetrates the flat region if and only if $\mu > 1/8$).



$r = 0$ singularity
i^+
$\mathscr{I}^+$
AH=EH
Schwarzschild
$\tau = \tau_2$
AH
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$\Sigma : \tau = \tau_\Sigma$
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i^0
$v = v_1$
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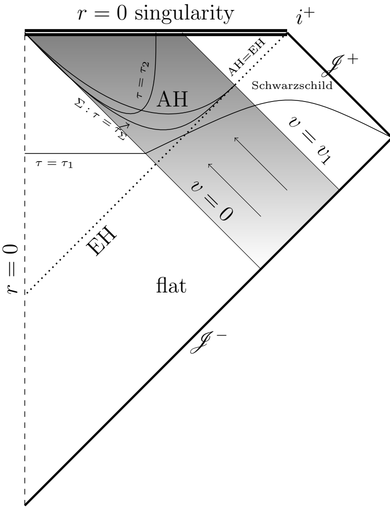

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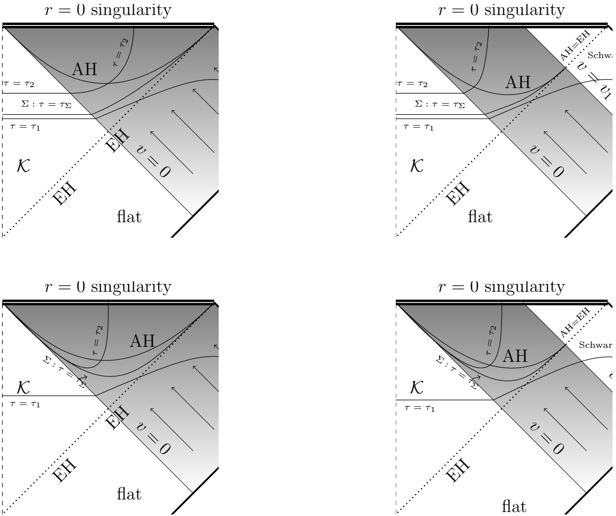
- *The minimum τ_m of τ on a closed f-trapped S is always attained within $I^{+}(AH)$ ($r < 2m(v)$)*

Proof:



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Furthermore, $\mathscr{B}$ cannot be non-spacelike everywhere; And it is spacelike close to its merging with Σ and EH.

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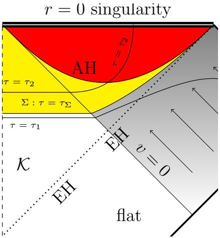
In particular,

$$\mathscr{B} \cap (\Sigma\backslash EH) = \emptyset$$


r = 0 singularity
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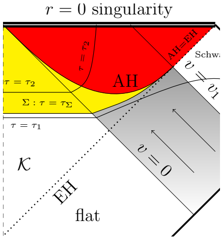
Σ : τ = τΣ
τ = τ1
EH
v = 0
K
EH
flat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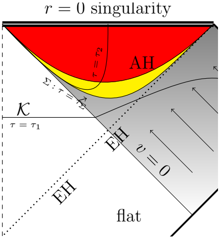
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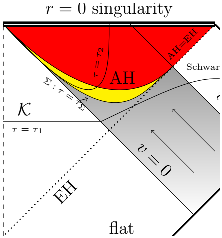
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Notice that the only closed marginally f-trapped surfaces that can be contained in $\mathscr{B}$ are those which are actually on its part $\mathscr{B} \cap$EH that coincides with the event horizon, if any.

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  - The Main Formula, the general theorems on minima, etc. are also general.